Can we work out Janssen equation parameters and stress dependence of the bulk density at low stress values during a simple uniaxial powder compression test?

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ABSTRACT

The vertical pressure profile of a powder column is usually described by the Janssen equation. Using uniaxial compression of polyethylene powders this was integrated assuming a linear dependence of the bulk density on the stress allowing to work out reasonable values for wall friction angle and lateral stress ratio.

INTRODUCTION

Powder compression is largely used in many industrial applications. This unit operation in mechanical process engineering is often preliminar to subsequent shearing to test the flowability of the powder. The pressure profile of a granular solid column in a cylindrical container is described using the well-known Janssen equation, which is often used even when additional surcharge stress is applied on the top of the column[1-3]. One of the controversial assumptions of this equation is the hypothesis of a constant bulk density. The significant pressure developed during compression and the simple fact that the volume decreases, even if only slightly, leads to experimental evidences that this conjecture cannot be made. One evidence, observed in this work, is the column height dependence of the measured "average bulk density". If a pressure dependence of the apparent density has to be effectively taken into account, then only an average density value can be inferred during uniaxial compression tests.

THEORETICAL BACKGROUND

The Janssen equation, deduced using the method of differential slices and a force balance on a discoidal element of volume in a cylindrical bin containing a cohesionless granular material, has the following form:

$$\frac{d\sigma_{zz}(z)}{dz} = -\frac{1}{\lambda}\sigma_{zz}(z) + g\rho \qquad (1)$$

where σ_{zz} is the normal stress along the column z axis (with origin at the center of the flat top surface), ρ is the granular material bulk density, g is the gravity acceleration and λ is a characteristic length which in turn is a function of the wall friction angle φ, of the container diameter D and of the Janssen coefficient K for the lateral stress transmission:

$$\lambda = \frac{D}{4Ktg\varphi} \qquad (2)$$

A solution for σ_{zz} can be analytically worked out by integration and is given by:

$$\sigma_{zz}(z) = \rho g\lambda\left(1 - e^{-z/\lambda}\right) + \sigma_0 e^{-z/\lambda} \qquad (3)$$

Here $\sigma_0=\sigma_{zz}(0)$ is a possible surcharge stress applied on the top of the column. As mentioned before this solution is worked out under constant bulk density conditions. In the following, some data will be shown for which this assumption clearly does not hold. In most cases, when a particular equation describing the stress dependence of the bulk density is chosen, the solution of the Janssen equation cannot be analytically found. Fortunately this is not the case when a linear pressure dependence of the bulk density is taken into account:

$$\rho(\sigma_{zz}) = a + b\sigma_{zz}(z) \qquad (4)$$

Here a represents the zero column height bulk density. The situation described by equation 4 cannot be always physically meaningful but at least at low stress values is justified by a first order approximation of any law for $\rho(\sigma_{zz})$.
The solution in this case can be expressed as follows:

$$\sigma_{zz}(z) = ag\Lambda\left(1 - e^{-z/\Lambda}\right) + \sigma_0 e^{-z/\Lambda} \qquad (5)$$

Which has the same form of equation 3 but with a different characteristic length given by the following equation:

$$\Lambda = \left(\frac{1}{\lambda} - bg\right)^{-1} \qquad (6)$$

Knowing a, b, φ and K it is possible to predict the new modified pressure profile and the resulting bulk density profile along the granular material column.

MATERIALS AND METHODS

Two industrially grounded linear low density polyethylene powders were used in this work. They are named in the following A and B.

Table 1. PSD of the two LLDPE powders.

	D(0,1) (μm)	D(0,5) (μm)	D(0,9) (μm)
A	177	389	786
B	303	571	1055

Their particle size distribution (PSD) characteristics are reported in table 1. The different PSD is due to different kind of mills employed.

Poured and tapped density of these two powders are also reported in table 2.

Table 2. Poured and tapped density of the two powders investigated.

	poured density (kg/m^3)	tapped density (kg/m^3)
A	420	470
B	320	390

In Figures 1 and 2 the morphology of the powders is shown in two optical microscope pictures.

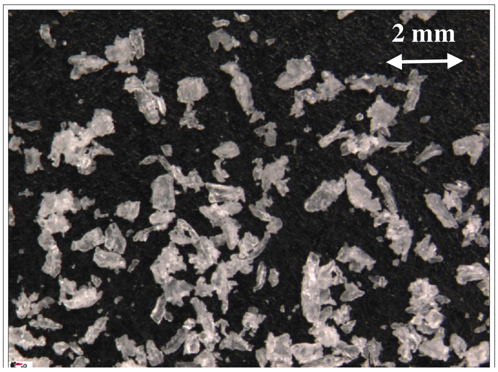

Figure 1. Morphology of powder A.

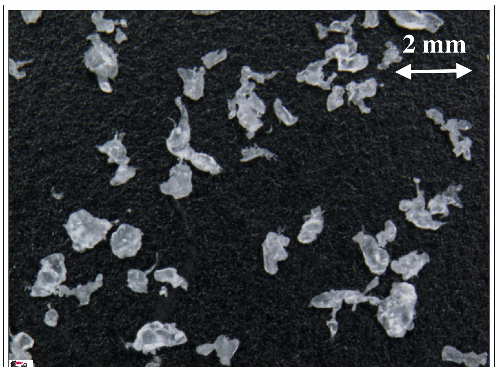

Figure 2. Morphology of powder B.

Uniaxial compression tests were performed using the cup of a Couette geometry of a TA Instruments rotational rheometer (ARES LSII with cup diameter=27 mm). A custom made upper cylinder was used to uniaxially compress the powders (see Figure 3). Three distinct measurements were performed by filling the cup with different amounts (and thus volumes V) of the same powder (column heights between 1 and 4 cm).

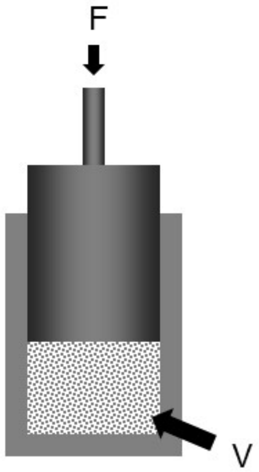

Figure 3. Cylindrical geometry used for uniaxial compression of the powders.

Using this kind of geometry and given the normal force F range provided by the instrument a maximum stress level of around 28 kPa was achieved.
Tapping experiments were performed using an Erweka tapped density tester.
A Brookfield Powder Flow tester (PFT) and an Anton-Paar MCR 702 rheometer equipped with the Warren-Spring geometry were also employed to determine wall friction coefficients for the two powders.

RESULTS AND DISCUSSION

In figure 4 the average bulk density as a function of the normal (surcharge) stress is reported as a typical example for powder A. The three initial column height values are also indicated. The bulk density is clearly not a constant and, considering states corresponding to the same normal stress value, a column height dependence of the average bulk density can be appreciated as highlighted in Figure 5.

In Figure 6 the normal stress dependence of the limiting zero-height bulk density is reported for sample A as a typical example.

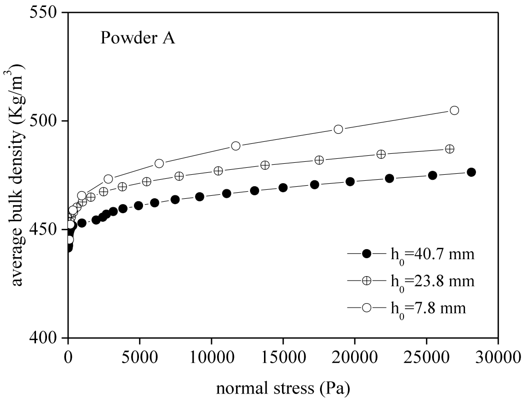

Figure 4. Dependence of the average bulk density on pressure for powder A.

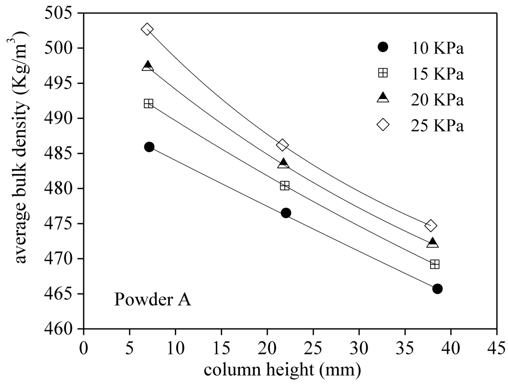

Figure 5. Dependence of the average bulk density on the column height for powder A at different stress levels.

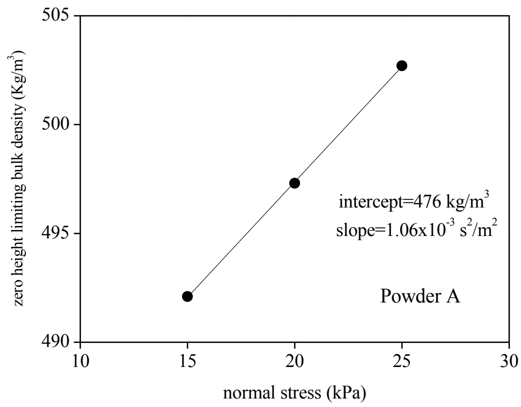

Figure 6. Dependence of the limiting zero-height bulk density on normal stress for powder A.

The values were extrapolated by fitting the data in Figure 5 using a simple exponential decay function as a first attempt. It can be seen that the assumption of a linear dependence is justified in the stress level range covered during the uniaxial compression performed by means of the rheometer.

Points in Figure 5 refer of course to an average bulk density:

$$<\rho>= \int_0^z \rho(z)dz \qquad (7)$$

Considering the linear dependence reported in equation 4 an attempt was made to simultaneously fit the experimental data reported in figure 5. The idea is that one can assign initial values for the four parameters a, b, K and $tg\varphi$ and then calculate the stress profile along the z axis. This immediately allows the evaluation of the bulk density profile along the column and then using equation 7 one can obtain the average bulk density value at each surcharge stress value for each filling level. The procedure ends when the mean square deviation between measured and predicted density values is minimized.

It has to be noticed that in all the equations the product $Ktg\varphi$ always appears. This means that it is not possible to calculate these two quantities independently. In Table 3 the best fit parameters for a, b and the product $Ktg\varphi$ are reported.

Table 3. Best fit a, b and $Ktg\varphi$ values.

	a (kg/m^3)	b (s^2/m^2)	$Ktg\varphi$
A	464	$2.47 \cdot 10^{-3}$	0.854
B	375	$1.97 \cdot 10^{-3}$	0.99

The values obtained for the "zero-stress bulk density" parameter a lie in the range between poured and tapped density values reported in Table 2. In addition this value practically coincides with the one obtained extrapolating at zero stress the zero column height trend of the bulk density reported in Figure 6. This makes sense and probably suggests that the number obtained this way is physically meaningful.

In addition the slope b is of the same order of magnitude as that obtained, always in figure 5, for the normal stress dependence of the extrapolated zero-height bulk density.

This latter finding further allows to consider the linear dependence proposed in Equation 4 a good physical assumption. This is also reasonable considering the low stress level involved in the uniaxial compression test performed using the rheometer (σ_{zz}<30 kPa). This stress level is suited for consolidation of the powder but very far from the values usually reached in compaction processes for tablets production.

In Table 4 the main results of wall friction tests performed on the two powders with a PFT are summarized.

Table 4. Friction angle, friction coefficient and fill density data obtained with a PFT.

	φ (°)	$tg\varphi$	ρ_{fill} (kg/m^3)
A	8.7	0.153	427
B	8.1	0.141	312

A comparison between the measured $tg\varphi$ data reported in Table 4 and the $Ktg\varphi$ data reported in Table 3 and obtained by the best fit of the experimental average bulk density profile measured during uniaxial compression allow to calculate the Janssen lateral stress ratio K for the two powders. This value is $K=5.5$ for powder A and $K=7$ for powder B.

In Table 5 the main results of wall friction tests performed on the two powders using a Warren spring geometry of a MCR 702 rheometer are summarized.

Table 5. Friction angle and friction coefficient data obtained with a MCR 702 equipped with the Warren spring.

	φ (°)	$tg\varphi$
A	12.8	0.227
B	11.1	0.196

In this case a comparison between the measured *tgφ* data reported in Table 5 and the *Ktgφ* data reported in Table 3 gave for the Janssen lateral stress ratio *K* for the two powders the values $K=3.8$ for powder A and $K=5$ for powder B.

All these *K* values, exceeding the usual range suggested for the active state ($0.3<K<0.6$), may suggest that a passive state of stress has to be taken into account for the uniaxially compressed powders at study.

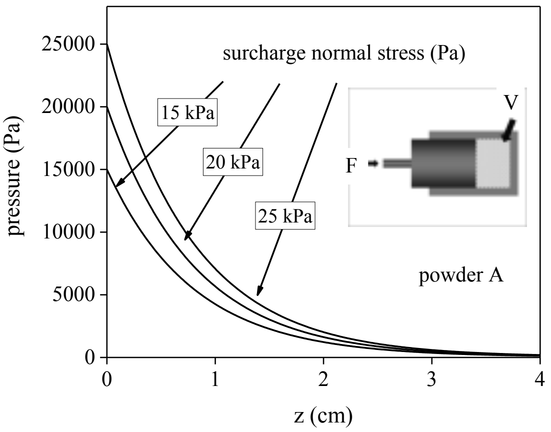

Figure 7. Axial pressure profile of powder A at different surcharge stress levels calculated using Equation 5.

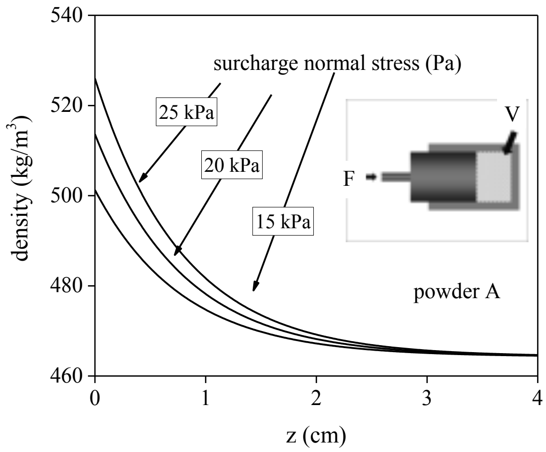

Figure 8. Bulk density axial profile of powder A at different surcharge stress levels calculated using Equation 5.

In figures 7 and 8 as a typical example the pressure and bulk density axial profiles for powder A are reported at different surcharge stress levels.

CONCLUSIONS

In this work an approach is proposed in which the linear dependence of the bulk density on the stress is assumed (low stress values) and the Janssen equation is consequently integrated analytically. Polyethylene powders were uniaxially compressed and their average bulk density values at different column heights and different stresses were modelled with the modified Janssen equation. This procedure allowed to work out reasonable values for the wall friction angle and the lateral stress ratio K in addition to the linear constitutive equation parameters describing the stress dependence of the bulk density.

Future additional work is recommended to validate the results obtained. For instance the number of points used to fit the average bulk density can be increased and wall friction coefficients could be calculated for the same kind of metal of the cup of the Couette. In addition assumptions concerning the fact that what is measured could not be a genuine principal stress needs further in-depth analysis.

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