## Oscillatory Shear Flows of Dense Suspensions of Elliptical Particles

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## **ABSTRACT**

Utilizing numerical simulations, we studied the rheological response of dense suspensions of elliptical particles under oscillatory shear and an imposed pressure. We compare aspect ratios 2 and 3. At large oscillatory strains, the oscillatory shear flows follow the Cox-Merz rule, while at low oscillatory strains, we get lower viscosities compared to the steady shear and higher shear jamming packing fractions. Thus far, the response is similar to the behavior of isotropic particles. However, unlike discs and spheres, elongated particles will end up in a dynamically locked state of their initial orientation at low oscillatory strains. This we could exemplify by simulating two different configurations with different nematic order parameters and the average orientation of the particles. Interestingly, the rheological response close to jamming in the frictionless case is not controlled by the average orientation and is instead coupled with the average number of contacts and the oscillatory strain. Having frictional contacts or larger oscillatory strains can help the system get out of these orientational arrested states, which at low strains evolve to a disordered state independent of the initial orientation, and at large strains develop into orientational ordered states.

## RESULTS AND DISCUSSIONS

We consider our two preparation protocols, pre-sheared and random packings which are initially at rest. Fig. 1, reproduced from our recent study, Yousefian and Trulson<sup>1</sup>, shows some typical numerical time-paths of the average direction angle  $\theta_{e\cdot\hat{y}}$  and the packing fraction  $\phi$  for oscillatory shear (OS) at a low oscillatory strain ( $\gamma_0 = 0.1$ ) and a selected J' (see Yousefian and Trulson<sup>1</sup> for the definition of  $\gamma_0$  and J') for both frictional (left panel) and frictionless (right panel) particles and for two different aspect ratios,  $\alpha = 2$  and 3. For the frictional cases both for  $\alpha = 2$  and 3, the two initial configurations converge to a directional disordered state with a low average orientation (and a low nematic ordering, see Yousefian and Trulson<sup>1</sup>) close to zero. This average direction angle is very close to the initial random configuration, *i.e.*, the pre-sheared samples relax to random configurations. The two configurations *i.e.* random and pre-sheared also converge to the same packing fraction higher than the corresponding steady-shear value for both of the investigated aspect ratios. Relaxation of the packing fraction is much faster than the relaxation of  $\theta_{e\cdot\hat{y}}$  (the number of contacts also relaxes quite rapidly and become identical for the two protocols, see Yousefian and Trulson<sup>1</sup>). Interestingly, for the frictionless ellipses, this is

not the case (see Yousefian and Trulson<sup>1</sup> for more details) and both  $\theta_{e\cdot\hat{y}}$  and  $\phi$  stay apart and no relaxation is detected withing our numerical time slab. The difference in the packing fractions of the two protocols is more observable for  $\alpha = 3$  (subplot (H)) than for  $\alpha = 2$  (subplot (F)). The gap, however, although diminished substantially, still exists for  $\alpha = 2$ . Moreover, the difference between  $\theta_{e\cdot\hat{y}}$ s of the two protocols is also smaller for  $\alpha = 2$  compared to  $\alpha = 3$ . Finding the  $\alpha$  threshold bellow which we lose these orientational arrested states requires further investigations.

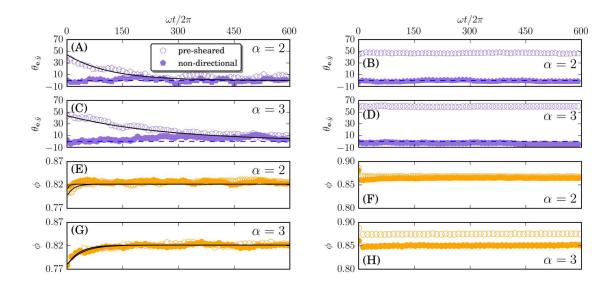


Figure 1: Time evolutions of  $(\mathbf{A},\mathbf{B})$  the orientation angle  $\theta_{e\cdot\hat{y}}(^{\circ})$  for  $\alpha=2$  (pentagons),  $(\mathbf{C},\mathbf{D})$  the orientation angle  $\theta_{e\cdot\hat{y}}(^{\circ})$  for  $\alpha=3$  (circles),  $(\mathbf{E},\mathbf{F})$  the packing fraction  $\phi$  for  $\alpha=2$  (pentagons) and  $(\mathbf{G},\mathbf{H})$  the packing fraction  $\phi$  for  $\alpha=3$  (circles) for configurations of elliptical particles with friction  $((\mu_p=0.4), \text{left panel})$  and without it  $((\mu_p=0), \text{right panel})$  at  $\gamma_0=0.1$  and  $J'\simeq 0.1$ . Pre-sheared and non-directional preparations are illustrated by open and full symbols, respectively. Black lines are best fits of the relaxations. The blue dashed lines in  $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D})$  represent zero degree.

To do a deeper exploration of the relaxation strains in the frictional and the frictionless cases, we estimate possible relaxation strains of various parameters with the exponential fit  $A(\gamma_{\rm acc}) = A_{\infty} + (A_0 - A_{\infty}) \exp(-\kappa \gamma_{\rm acc})$ , where A can be either of  $\theta_{e \cdot \hat{y}}$  or  $\phi$ , and  $\kappa^{-1}$  represent a relaxation strain (see Yousefian and Trulson<sup>1</sup> for more details). The exponential description of the relaxations are shown by solid black lines in the frictional cases in Fig. 1. According to Yousefian and Trulson<sup>1</sup> that direction parameters like the nematic ordering of the particles  $S_2$  and  $\theta_{e\cdot\hat{y}}$  generally follow each other in terms of relaxation strains (see Fig. 1(a) and (c) in Yousefian and Trulson<sup>1</sup> and the supplementary information of  $^{1}$ ), while  $\phi$  and the number of contacts Z relax an order of magnitude faster than the orientational properties (see Fig. 1(e) and (q) in Yousefian and Trulson<sup>1</sup>). The relaxation strains depend on friction coefficient, oscillatory strain (see SI of Yousefian and Trulson<sup>1</sup>), and J'. If however, the system is frictionless at a small  $\gamma_0$  (e.g., 0.1), it shows an infinite orientational relaxation strain. The packing fraction relaxes almost instantaneously, but to different values depending on if one starts from a random or pre-sheared configuration (see right panel in Fig. 1). In our recent study, Yousefian and Trulson<sup>1</sup>, we did further investigation to find the friction coefficient threshold bellow

which we observe orientational arrest at low  $\gamma_0$  (see **Fig. 2** in Yousefian and Trulson<sup>1</sup>) and we found that the threshold must lie somewhere between  $\mu_p = 0.1$  and  $\mu_p = 0$ . Since,  $\mu_p = 0.1$  was the lowest friction coefficient for which we could obtain the slowest detectable relaxation rate, the exact value of this friction threshold remains as an open question. Similarly,  $\gamma_0$  needs to be less than 0.3 for the arrested states and accordingly infinite relaxations to occur (see SI of Yousefian and Trulson<sup>1</sup>).

We also investigated how the complex viscosity varies with packing fraction, and  $\gamma_0$ for our two preparation protocols compared to frictional particles (see Fig. 4 (a,b), in Yousefian and Trulson<sup>1</sup>) considering that the frictionless particles do not fulfil one unique equation of state for low oscillatory strains both for  $\alpha = 3$  and  $\alpha = 2$ . For both frictional and frictionless ellipses, we find that the rheological response (i.e.,  $|\eta^*|$  vs.  $\phi$ ) behave as their corresponding steady shear (SS) cases for large  $\gamma_0$ s. At lower  $\gamma_0$ s, we get a lower viscosity than SS at the same packing fraction, with an increased shear jamming packing fraction for the frictional particles i.e.,  $\phi_{c,f}^{SS} < \phi_{c,f}^{OS}$ . These findings are in line with findings of Dong and Trulson<sup>2</sup> for discs. Unlike for isotropic particles, frictionless ellipses at low  $\gamma_{0}$ s show a preparation dependent rheology (see Fig. 4 in Yousefian and Trulson<sup>1</sup>), with higher shear jamming packing fractions for pre-sheared preparations compared to nondirectional/random ones, i.e., elongated frictionless particles at low  $\gamma_0$ s have a viscosity of the form  $\eta^*(\gamma_0, \phi, initial \theta_{e \cdot \hat{y}})$  while in the disc case, Dong and Trulson<sup>2</sup>, it is only  $\eta^*(\gamma_0,\phi)$ . Nonetheless, once the complex viscosity  $|\eta^*|/\eta_f$  is plotted against the number of contacts Z in Fig. 4 (a,b) of Yousefian and Trulson<sup>1</sup>, for frictionless particles, the two protocols with different  $\phi$ 's collapse on each other despite having both different packing fractions and orientational properties, i.e., the viscosity for frictionless ellipses at low  $\gamma_0$ s can be alternately described as  $\eta^*(\gamma_0, Z)$ .

## **REFERENCES**

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