

## Reflections over Metzner's method

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### ABSTRACT

Tube and mixer flows are central in the process industry. For dimensioning of non-Newtonian flow systems, Metzner's method is often used but it sometimes also occurs in rheological connections. Its basic meaning does, however, not always seem to be clear. Its application for fibre suspension tube flow has been criticised, by e.g. Duffy, who points out that it means a correlation between the friction factor and itself. In this work, various aspects of this method are discussed. It is shown that the problems are more general, and that the method in some cases may be simplified.

### INTRODUCTION & LITERATURE

There are two main problems in rheology. The first and more theoretical, concerns how to describe the flow properties. The second, more practical, concerns the problem of measuring them, i.e. rheometry. A central task in theoretical rheology is to explain the flow behaviour in terms of flow mechanisms, i.e. a scientific problem. Another task is to predict the behaviour in various equipments when the rheology is known, i.e. an engineering problem. The prediction problem can be approached in two ways: 1) to adapt the rheology to a model that is mathematically manageable, 2) to use the measured rheology directly. In the present work, the latter approach will be discussed.

These methods were developed to meet specific demands in the industry, and their development can be described as a relay run between transport systems (tube flows) and mixer systems. The formative literature will

first be shortly reviewed. In next section, the two variants will be discussed consecutively.

The first work came from industry and Karl Magnusson<sup>1</sup> who in 1952 (in Swedish) considered the problem of dimensioning the engine of a mechanically stirred mixer. Its purpose is initially difficult to grasp even for a Swede due to the title, which means "The dependence of the power consumption upon the efficient viscosity in mixing structure-viscous liquids" when it actually describes how to practically evaluate efficient viscosities  $\mu_{eff}$  (*effektiva viskositeter*) from the Newtonian and non-Newtonian power curves  $P(N)$  obtained in the same mixer (here blade diameter and liquid height 35 cm, blade width 17.5 cm, blade height 13.3 cm, and bottom clearance 6 cm, filled with a non-Newtonian liquid of 59% zinc white and 41% raw linseed oil for paint). Magnusson refers to similar evaluations in tubes made earlier by other. The thus calculated  $\mu_{eff}$  values were finally compared with a relative viscosities value obtained with a Wolf-Hoepke "turbo viscometer" (*turboviskosimeter*), i.e. a mixer of diameter 8 cm, liquid height 3 cm, S-formed blade, and primary data being the weight needed to rotate it at 90 rpm, which was transferred to "viscosities" through calibration with a Newtonian fluid. Magnusson's work is characterised by an uncertain use of rheological concepts, e.g.  $\mu_{eff}$  used as the same as the rheological apparent viscosity (*skenbara viskositeten*),  $\mu_a \equiv \tau/\dot{\gamma}$  and both measures are compared directly with the Wolf-Hoepke viscosity.

No rheological ambiguities prevail in next often-cited work, by Bengt Hedström<sup>2</sup> at KTH, also from 1952. It deals with the opposite approach, *viz.* the calculation of pipe pressure drop curves for a Bingham fluid  $\tau = \tau_0 + \mu_p \dot{\gamma}$ . It presents nothing rheologically new compared Buckingham's solution from 1921. Hedström's contribution instead consists of a dimensional analysis, and construction of graphical solutions with help of nomograms and, more interesting here, the introduction of a modified Reynolds' number  $N_{Re} = \rho VD/\mu_p$  (later bearing his name).

The third central work came in 1954 from A.B. Metzner at Colgate-Palmolive<sup>3</sup>. Like Magnusson's work, it aims at a general approach, and e.g. paper pulp suspensions and heterogeneous slurries are especially mentioned. Also here a rheological vagueness prevails. In addition, two debatable conjectures are forwarded. Firstly, that the Poiseuille equation can also be made applicable for laminar flow of a non-Newtonian fluid by substitution of a judiciously chosen *apparent viscosity*  $\mu_a$ , for the Newtonian viscosity  $\mu$ . To be non-trivial, this ought to aim at more than an experimental adaptation factor like  $\mu_{eff}$ . Secondly, the conjecture that the (unspecified) shear rate  $\dot{\gamma}_t$  in tubes (diameter  $D$ , flow velocity  $V$ ) is a unique function of  $V/D$ , say  $f(V/D)$ , that the (unspecified) shear rate  $\dot{\gamma}_v$  in a (not specified) rotational viscometer depends on the rotational speed  $N$  through *the same* unique function, i.e.  $f(N)$ , and that the ratio between the two shear rates is a unique function of  $V/DN$ , i.e.  $\dot{\gamma}_t / \dot{\gamma}_v = f(V/D)/f(N) = g(V/DN)$ . A comparison between the straight-lined rheogram and Buckingham's non-linear solution, in e.g. Hedström,<sup>2</sup> is sufficient to show that this cannot be generally valid. Through series development of  $f$  and  $g$  it may also be shown that  $f(x)/f(y) = g(x/y)$  applies generally for potential functions of the same order, i.e. in rheology for Ostwald-

deWaele fluids (without a yield stress and including Newtonian fluids). Based on this, a new dimensioning method was proposed, which has been found to work.

The rheological terminology was straightened out a year later in Metzner and Reed,<sup>4</sup> where the ambition was to give the method a solid theoretical basis. Starting with the Mooney-Rabinowitsch equation (*MRE*), generalised viscosities  $\mu_{gen}$  and generalised Reynolds' numbers  $N_{Re,gen}$  were introduced. These were recommended to be obtained from tube flow or tube rheometer experiments. It is stated that the method, like *MRE*, is applicable only for fluids without wall-slips, i.e. a restriction compared to before. Since this is impossible to know in advance, the authors state that it should be checked in advance, e.g. in tubes of different diameters. Nevertheless, they present literature results in support of their method for e.g. 4% paper pulp suspension where this definitely does not hold (as shown e.g. by Duffy, the author and many other in fibre flow branches) but the method still works.

In Metzner and Otto<sup>5</sup> from 1957, Magnusson's ideas are developed. Here a rheological uncertainty partially returns. E.g., the *efficient viscosity* is named *apparent viscosity*, which is also used to designate the rheological *apparent viscosity*. It is further assumed that an (unspecified, but probably volumetric) average shear rate exists that "around the impeller" and that this is linearly dependent on the stirrer speed, i.e.  $\dot{\gamma}_{average} = kN$ . The validity of this is tested in Metzner and Taylor<sup>6</sup> from 1960 through photographic flow tracer studies. Finally, in Metzner *et al.*<sup>7</sup> from 1961, a larger investigation is carried out to determine  $k$ -values for different mixers. Somewhat earlier, a similar investigation had been started by Calderbank and Moo-Young.<sup>8,9</sup> The method can then be said to be established and went into important monographs like Skelland,<sup>10</sup> Aiba *et al.*,<sup>11</sup> etc.

The author met the method in 1978 in a course based on Skelland,<sup>10</sup> next in

biotechnology in connection with the rheology of mycelial fibre suspensions.<sup>12</sup> Two principally different ways exist of dealing with the unavoidable heterogeneity-caused signal noise in these porridge-like fluids. One is to let the flow system of the rheometer effectuate the averaging. This approach has just been presented in the literature with so-called turbine rheometry based on Metzner's method.<sup>13</sup> The author had tested it but found it enough disquieting to rely on a method that was impossible to understand intuitively to carry out the mixer analysis below. As a result, turbine "rheometry" was abandoned in favour the opposite approach, i.e. to simplify the flow field as much as possible.<sup>12</sup> When later, higher shear rates should be reached, tube rheometry became natural.<sup>14</sup> Inspired by research in paper technology, the author built a tube circuit, which was used as a rheometer, instead of as by them for tube flow studies *per se*. The author was then back in Metzner's method, but could not then see the connection with the earlier used mixer version. However, it was difficult to e.g. understand that the results became less scattered with the coarse fibre suspension than for the smooth calibration fluids, since this was contrary to his experience with cylinder rheometry. The connection was then realized and the analysis presented in next section made. As a result, also this approach was left. The whole method had almost been repressed when Duffy's articles<sup>15</sup> started to appear in the late 1990's. He had experimentally concluded that the method basically meant a correlation of the friction factor with itself. This was discussed at KTH in the late 1990's. Duffy regarded it as a fibre flow and misapplication problem,<sup>16</sup> the author also as more general.

## METZNER'S METHOD

### Newtonian transport systems

Consider first a Newtonian tube flow system described by the implicit function

$$F(\Delta P/L, V, \mu, \rho, D) = 0, \quad (1)$$

where  $\rho$  is the liquid density and  $\mu$  the viscosity. Buckingham's  $\Pi$ -theorem shows that it may be reformulated as

$$F(N_F, N_{Re}) = 0 \quad \text{or} \quad N_F = G(N_{Re}), \quad (2, 3)$$

with the dimensionless Reynolds' number

$$N_{Re} = \rho V D / \mu, \quad (4)$$

and the dimensionless Fanning friction factor

$$N_F = (D \Delta P / 4L) / (\rho V^2 / 2), \quad (5)$$

where  $\Delta P/L$  is the pressure gradient along the tube. When inertia, represented by  $\rho$ , vanishes a series development of Eq. 3 gives

$$N_F \propto A / N_{Re}. \quad (6)$$

On the other hand, under pure inertial conditions, i.e. when the influence of  $\mu$  is negligible (which eventually may never occur in practice), the development gives

$$N_F = B. \quad (7)$$

The constants  $A$  and  $B$  can be obtained experimentally or theoretically. Theoretically, a comparison with Hagen-Poiseuille's equation gives  $A = 16$ , i.e.

$$N_F = 16 / N_{Re} \quad (8)$$

for  $N_{Re} < 2100$ , giving a straight line with slope  $-1$  in Moody's double-logarithmic diagram. For moderately high  $N_{Re}$ , the semi-empirical Blasius curve is normally displayed. The use of this diagram is too well known for further consideration here.

*Reflection 1.* Duffy's central criticism concerns that mechanistic knowledge cannot be ignored, if the risks of wrong conclusion should be eliminated. To see this, it is again easiest to start with a Newtonian system and the question what information can be

obtained if *only* pressure drop  $\Delta P/L$  at different volumetric average velocities  $V$  can be measured, i.e. without the possibility to e.g. observe the flow. Assume therefore that the experiment show that

$$\Delta P/L \propto V. \quad (9)$$

Hagen-Poiseuille's equation, i.e. the explicit form of Eq. 8, reads

$$\Delta P/L = 32\mu V/D^2 \propto V. \quad (10)$$

Comparison with the experimental result therefore according to Eq. 9, implies a Newtonian flow. But then consider a plug tube flow, of e.g. paper pulp, with a narrow Newtonian wall-layer of constant width  $\delta$ . Thus, with  $V_{plug} \approx V$ , the wall shear rate becomes  $(V_{plug} - 0)/\delta \approx V/\delta$ . An axial balance of the shear forces  $(\mu V/\delta)(\pi DL)$  and total pressure drop force  $\Delta P \pi D^2/4$  gives

$$\Delta P/L = 4\mu V/D\delta, \quad (11)$$

i.e. also  $\propto V$ . If it is assumed that the flow is Newtonian, identification with Eq. 10 then indicates a Newtonian flow with viscosity  $\mu D/8\delta$ , which would be large because of the smallness of  $\delta$ . If such a substance (e.g. a paper pulp suspension), would be observed in e.g. a beaker, it would probably have a gel-like appearance, and one would not see anything strange in the high viscosity value. It would thus be difficult from just the rheological measurements to suspect anything else but homogeneous and Newtonian. Duffy's criticism thus also applies the simplest of cases. A *substance* may be homogeneous *per se*, but the presence of wall-layers anyhow makes the *system* heterogeneous. And every substance has a wall-layer, just because the wall region is different from the bulk region, although the effects can sometimes be neglected. Sometimes the wall region is all, as in Knudsen flow. Duffy's criticism therefore

also applies non-fibrous "continuous" substances.

It may be suggested that if the plug flow does not differ from Newtonian behaviour, it does not matter if it is also treated as Newtonian. The problem is that it does. A doubled tube diameter would e.g. for the Newtonian flow give a pressure drop decrease to 1/4th, whereas for plug flow just to 1/2. The perfect rheologist would exclude this by initially studying differences between tubes of different diameters, and if these are found, subtract the wall-effects (e.g. with Mooney's method) before evaluating the rheogram. The point is that this is easier said than done, especially routinely, see e.g. Björkman<sup>7</sup> for fibre flow.

The roots of these problems can be described in many ways. Duffy emphasises the special character of fibre suspensions. It is true that fibre plugs and flocs are exceptionally difficult to break down in comparison with many other due to the extremely spread-out mass distribution in fibres,<sup>12</sup> but according to the author this is more a quantitative difference, Björkman.<sup>17</sup> Instead he prefers to see the problem as caused by the fact that the flow units, in this case the plug, are too large compared to the tube diameter to make a continuum approach relevant for the entire system consisting of the tube, the wall-layer and the plug.

The roots of the problems may also be expressed dimensionally. That is, that the inclusion of a further parameter is necessary to make the system description Eq. 1 complete, e.g. the floc (i.e. the plug) diameter  $D - 2\delta$ , which dimensionally is equivalent to  $\delta$ . This would result in e.g.

$$N_F = G(N_{Re}, \delta/D), \quad (12)$$

in place of Eq. 3, which in turn may be series developed, etc. as before.

#### Non-Newtonian transport systems

The goal of Metzner and Reed<sup>4</sup> was to extend the  $N_F - N_{Re}$  approach to non-Newtonian tube flow systems.  $N_F$  contains

no rheology, but in  $N_{Re}$  it enters in the viscosity term. The problem therefore concerns how to define the viscosity for a non-Newtonian fluid when the shear rate varies throughout the tube – from zero at the axis to maximum at the wall. They started with  $MRE$ , which gives the wall shear rate

$$-\dot{\gamma}_w = [(3n'+1)/4n'](8V/D), \quad (13)$$

where

$$1/n' \equiv d(\ln 8V/D)/d(\ln D\Delta P/4L) \quad (14)$$

is obtained from the pressure drop curve  $P(V)$ . If  $n'$  is constant, Eq. 14 may be integrated to

$$D\Delta P/4L = K'(8V/D)^{n'}. \quad (15)$$

$MRE$  presumes no wall-slip, which was clearly stated. They realised that Eq. 15 could be brought on the Hagen–Poiseuille form Eq. 8, or  $N_F = 16/N_{Re}$ , if one defines a generalised Reynolds' number

$$N_{Re,gen} \equiv \rho D^n V^{2-n'} / \mu_{gen}, \quad (16)$$

with the generalised viscosity  $\mu_{gen}$  of the non-Newtonian fluid defined by

$$\mu_{gen} \equiv 8^{n'-1} K'. \quad (17)$$

They also noted that with  $n'=1$ , all expressions transforms correctly to the Newtonian expressions if letting  $K = \mu$ .  $K'$  and  $n'$  were related to the Ostwald-deWaele coefficients in  $\tau = K\dot{\gamma}^n$ , by letting  $\dot{\gamma}_w = \dot{\gamma}$ , giving  $n = n'$  and  $K = K'[4n'/(3n'+1)]^n$ . Hence, it would be possible to present data for both Newtonian and non-Newtonian fluids in the same type of diagram.

Based on this, they suggested the following procedure for calculating the pressure drops  $\Delta P/L$  at flow rate  $V$  in tubes of diameter  $D$ . Firstly,  $K'$  and  $n'$  are evaluated for the non-Newtonian fluid –

preferably in pipes, with a tube rheometer or from the literature (less likely to become so generally available as viscosity for Newtonian fluids). When  $K'$  and  $n'$  are known,  $\mu_{gen}$  can be evaluated with Eq. 17. With  $\rho$ ,  $\mu_{gen}$ ,  $V$  and  $D$  at hand Eq. 16 gives  $N_{Re}$ . Then,  $N_F$  may be obtained either graphically from the Moody-diagram, or numerically with the Hagen-Poiseuille formula Eq. 8. Finally, when  $\rho$ ,  $V$ ,  $D$  and  $N_F$  are known, the definition of friction factor Eq. 5 gives  $\Delta P/L$ .

*Reflection 1.* Under *inertial* conditions they recommended the  $K'$  and  $n'$  evaluated under *non-inertial* conditions to be used, i.e. in the same way the "non-inertial" Newtonian viscosity  $\mu$  is used in the ordinary Reynolds' number also under turbulent conditions. As a result, problems may occur if  $K'$  and  $n'$  not are constant *and* cannot be measured under non-inertial conditions for the  $8V/D$  at which e.g.  $\Delta P/L$  should be evaluated. The Newtonian correspondence to this dilemma would be to not have an enough narrow tube at hand to reach laminar conditions under the smallest available pumping rate to measure a "laminar" Newtonian viscosity.

*Reflection 2.* The efficient viscosity is defined by Eq. 10,  $\mu_{eff} = (\Delta P/L)/(32V/D^2)$ . Insertion in the previous definitions and equations shows that  $\mu_{gen} = \mu_{eff} (V/D)^{1-n'}$ , i.e.  $\mu_{gen}$  and  $\mu_{eff}$  are closely related. Insertion of this in Eq. 16 leads to the same conclusions as under next point.

*Reflection 3.* That a limiting behaviour of something occurs correctly does not automatically means that it is *physically* meaningful. The approach of  $N_F = 16/N_{Re}$ , in the Newtonian limit can be obtained in many ways, e.g. with  $N_{Re,eff} = \rho D V^2 / \mu_{eff}$  and a correlation  $\mu_{eff} = K''(V/D)^{n''}$ , without mathematical need to keep  $K''$  and  $n''$  constant (but they are of course constant to the same degree as  $K'$  and  $n'$  through the relation between  $\mu_{gen}$  and  $\mu_{eff}$  given above). The complications (dimensional, etc.) are thereby kept within one term instead of

three, due to a somewhat arbitrary, start with *MRE*. Hereby, the basic meaning of the method stands out clearer.

*Reflection 4.* In the non-inertial regime,  $\Delta P/L$  can always be obtained with less labour than the suggested. To calculate the generalised  $N_{Re}$  it had e.g. not been necessary to explicitly evaluate  $\mu_{gen}$ . If namely its definition is inserted one obtains

$$N_{Re} = 8\rho V^2/K'(8V/D)^{n'}, \quad (18)$$

which, may be calculated for wanted  $V$  and  $D$  when  $\rho$ ,  $K'$  and  $n'$  are known. One could also have inserted this algebraic expression in the Hagen-Poiseuille formula, Eq. 8, and then with the definition of  $N_F$  obtained the original tube pressure drop expression

$$D\Delta P/4L = K'(8V/D)^{n'}, \quad (19)$$

which equally well can be used directly to get the pressure drop at the desired  $V$  and  $D$  when  $K'$  and  $n'$  are known. With the last expressions for  $N_{Re}$  and  $D\Delta P/4L$  one gets

$$N_{Re} = (16\rho V^2/2)/(D\Delta P/4L) = 16/N_F, \quad (20)$$

which with the Hagen-Poiseuille formula Eq. 8 is equal to  $N_{Re}$ , and as Duffy<sup>15</sup> correctly has observed through experiments.

*Reflection 5.* The power consumption per unit volume  $p$  is  $V\Delta P/L$ . Then Eq. 15 gives  $p = (K'/2)(8V/D)^{n'+1} \equiv M(8V/D)^m$ . If, in line with *MRE*,  $m$  is allowed to vary with  $8V/D$ , one obtains  $p = M(8V/D)^{m(8V/D)} \equiv f(V/D)$ . The method thus basically expresses that the average dissipation  $\Phi_v = p$  is a function of  $V/D$ , without involving any rheology, i.e.  $\Delta P/L = 4\Phi_v(V/D)/V$ . Inclusion of the flow properties in method therefore must become tautological, which is not wrong *per se*. Mathematics often has this character. The question is if the extra complications and labour are practically motivated.

### Non-Newtonian mixing systems

For mixers, the impeller diameter  $d$  is normally used as characteristic length, and the rotational speed  $N$  as characteristic kinematical measure. Dimensional analysis of the implicit system description

$$F(P, N, d, \rho, \mu) = 0 \quad (21)$$

shows that it may be reformulated as

$$F(N_P, N_{Re}) = 0 \quad \text{or} \quad N_P = G(N_{Re}), \quad (22, 23)$$

with the power number given by

$$N_P = P/\rho N^3 d^5 \quad (24)$$

and the modified Reynolds' number by

$$N_{Re} = \rho N d^2 / \mu. \quad (25)$$

The series development of Eq. 23

$$N_P = \Sigma A_n N_{Re}^n, \quad (26)$$

for non-inertial conditions gives

$$N_P = A/N_{Re} \quad (27)$$

and for purely inertial conditions

$$N_P = B. \quad (28)$$

Contrary to with tubes, the constants  $A$  and  $B$  can, due to the complex flow geometry, seldom be calculated. The problem with non-Newtonian mixing is again confined to what viscosity  $\mu$  to use in  $N_{Re}$  since it varies with deformation rate and therefore also throughout the mixer.

The complete mixer method in detail contains the following steps.

1. With the mixer filled with a Newtonian fluid and in scale  $d$ , the Newtonian power curve  $P_N(N, d)$  is measured.
2. With  $\rho$  and  $\mu$  known, the Newtonian  $N_P(N_{Re})$  diagram is calculated with Eqs. 24 and 25.

3. With the same mixer filled with the non-Newtonian fluid, the non-Newtonian power curve  $P_{n-N}(N, d)$  is measured.
4. With  $\rho$  known for the non-Newtonian fluid, Eq. 24 gives  $N_{P_{n-N}}(N)$ .
5. With the help of the Newtonian  $N_P(N_{Re})$  curve, a non-Newtonian Reynolds' number  $N_{Re\ n-N}(N)$  is evaluated.
6. With  $\rho$  and  $d$  known, a non-Newtonian viscosity  $\mu_{n-N}(N)$  is calculated with Eq. 25.
7. The non-Newtonian viscosity is set equal to an apparent viscosity, i.e.  $\mu_a = \mu_{n-N}$ .
8. The rheogram  $\tau(\dot{\gamma})$  of the non-Newtonian substance is determined with a rheometer.
9. This rheogram is used for assigning an apparent shear rate, i.e.  $\dot{\gamma}_a = \tau(\dot{\gamma}_a)/\mu_a$ .
10.  $\dot{\gamma}_a$  is plotted against  $N$ . The slope gives the constant  $k$  in  $\dot{\gamma}_a = kN$ .
11. The apparent shear rate in the scaled non-Newtonian system, defined by  $d' = \alpha d$  and  $N' = \beta N$ , is estimated as  $\dot{\gamma}_a' = kN'$ .
12. The apparent viscosity of the scaled non-Newtonian system becomes  $\mu_a' = \tau(\dot{\gamma}_a')/\dot{\gamma}_a'$ .
13. The non-Newtonian viscosity is set equal to the apparent viscosity in the scaled system, i.e.  $\mu_{n-N'} = \mu_a'$ .
14. The scaled non-Newtonian Reynolds' number is set to  $N'_{Re, n-N} = \rho N' d'^2 / \mu'_{n-N}$ .
15. The Newtonian  $N_P(N_{Re})$  curve is used to evaluate  $N_{P\ n-N'}(N')$ .
16. Eq. 24 finally gives  $P_{n-N'} = \rho N'^3 d'^5 N_{P\ n-N'}$ .

Some of the above steps were motivated physically. Thus, step 5 to 7 was said to guarantee a correct behaviour in the Newtonian limit. Step 10 was tested experimentally with photographic flow tracer studies, Metzner and Taylor.<sup>6</sup> It was there called an average shear rate in the impeller region, but used in the above context.

Since it is difficult to intuitively grasp the overall meaning of all steps, the individual steps will first be commented on. Step 1 & 2. The meaning of is to determine the constant  $A$  in relation Eq. 27. This constant is not unequivocally related to the

non-Newtonian power curve. Although both systems have the same boundary conditions, the differential equations due to the constitutive relations are different. The non-Newtonian case shall, however, of course be approach the Newtonian system in the limit. Step 3 is an uncontroversial experimental determination of  $P_{n-N}(N, d)$ . Step 4, 5 & 6 mean that  $P_{n-N}(N, d)$  is divided by the factor  $AN^2d^3$ , i.e. an assignment of an efficient viscosity  $\mu_{eff}$ , which is the viscosity a Newtonian fluid should have to give the same power  $P$  at certain rotational speed  $N$ . It is also difficult to give  $\mu_e$  a direct *physical* meaning.

Step 7. To let  $\mu_a = \mu_{eff}$  is a purely formal step, since it is generally meaningless to speak about the rheology without reference to kinematics and flow geometry. These are different in a mixer and a true rheometer.

Step 8 & 9. The rheogram for the non-Newtonian fluid is used to convert  $\mu_a$  in previous step to an apparent shear rate  $\dot{\gamma}_a$ . Rheology can be described in many ways, the most common being the rheogram  $\tau = f(\dot{\gamma})$ , or the flow curve  $\dot{\gamma} = f^{-1}(\tau)$ . But also the apparent viscosity  $\mu_a = \tau/\dot{\gamma} = f(\dot{\gamma})/\dot{\gamma} \equiv F(\dot{\gamma})$  may be used, or its reverse  $\dot{\gamma} = F^{-1}(\mu_a)$ . Thus,  $\dot{\gamma}_a = F^{-1}(\mu_a) = F^{-1}(P(N, d)/AN^2d^3)$ .

Step 10. That  $\dot{\gamma}_a$  is linearly proportional to  $N$  was tested experimentally for some fluids.<sup>6</sup> It is, however, difficult to see that this should be generally valid. To not rule out the method for this, also a non-linear  $\dot{\gamma}_a(N)$  will be allowed, including the linear case.

Step 11 means a kinematical scaling in  $N$ , i.e.  $\dot{\gamma}_a' = F^{-1}(P(N', d)/AN'^2d^3)$ . Implicit is that  $\dot{\gamma}_a(N)$  is independent of the length scale.

Step 12 and 13 mean  $\mu_{n-N'} = \mu_a' = F(\dot{\gamma}_a') = F(F^{-1}(P(N', d)/AN'^2d^3))$ .

Step 14, 15 & 16 reverse the first two steps, which totally gives,  

$$P'(N', d') = AN'^2d'^3\mu_{n-N'} = AN'^2d'^3F(F^{-1}(P(N', d)/AN'^2d^3)) =$$

$$= AN^2 d^3 P(N', d) / AN^2 d^3 = \alpha^3 P(\beta N, d).$$

The overall meaning of the method thus is  $P/V \equiv p = \Phi_v(N)$ . The 16 steps are the same as:

1. Determine the non-Newtonian  $P(N, d)$  experimentally.
2. Scale according to  $P' = \alpha^3 P(\beta N, d)$ .

$P/V = \Phi_v(N)$  thus plays the role of  $D\Delta P/4L = K'(8V/D)^{n'}$  for the tube flow.

The documentation needed for the full method is  $\dot{\gamma}_e(N)$ , or the slope if the relation is linear, and the rheology e.g. as for  $\tau(\dot{\gamma})$ . For the two-steps method only the average dissipation rate  $\Phi_v(N)$  is needed.

*Reflection 1.* With the full method, we perform the Newtonian and the non-Newtonian power measurements, and need a rheometer to determine the rheogram (the method will, however, give correct results also with an arbitrary function instead of the physically based constitutive relation). As the method is used, the rheology is superfluous. The information utilised is contained in the non-Newtonian power curve  $P_{n-N}(N, d)$ .

*Reflection 2.*  $k$ -values for different mixer geometries are tabulated in the literature,<sup>10</sup> e.g.  $11.5 \pm 1.5$  for Rushton tanks. Using  $k$ -values to calculate  $P(N)$  for a new fluid proceeds in the following way:

1.  $\tau(\dot{\gamma})$  is determined experimentally,
2.  $\dot{\gamma}_e = kN'$ ,
3.  $\dot{\gamma}_e$  and  $\tau(\dot{\gamma})$  gives  $\mu_e$ ,
4.  $N_{Re} = \rho N d^2 / \mu_e$ ,
5. The ordinary power curve then gives  $N_P$ , and
6.  $P = N_P \rho N^3 d^5$ .

The success of this method thus depends on that  $k$  does not vary more than the precision demands of the predictions (say 10%). This is the same as saying that the average dissipation rate  $P/V = \Phi_v(N)$  is sufficiently equal for different rheologies.

*Reflection 3.* A central question concerns that parameters with the same dimension may have different physical background.

They may be completely unrelated, as e.g. kinetic and potential energy, or partly related, as kinetic and total energy, or as here the relation between apparent and efficient viscosities. This especially applies  $\mu_{gen}$ , which (as used) relates more to power consumption rather than viscosity.

*Reflection 4.* At the core of Metzner's method, both its transport and mixer variants, seems to hide scaling based on average dissipation rate-based. This may be as good approach as any, but the question is, if it is necessary to involve the rheological apparatus for this, especially using rheological concepts like e.g. generalised viscosities and Reynolds' numbers that basically not are rheological, but more of energetic/ thermodynamic origin.

*Reflection 5.* Some practical and principal differences between and tube and mixer flow, however, exist. Tube flow is mainly used for transport, and not mixing, whereas mixing is used mainly for mixing. This means that tubes may operate under non-inertial, transition or inertial conditions. Mixers, however, starts to work efficiently firstly when secondary flow starts, i.e. under transitions and inertial conditions, but more seldom under non-inertial conditions (disregarding very special kind of mixers like kneaders, etc.).

## CONCLUSION

The method discussed in this work is, and was not intended, to be more than an approximate engineering method. It seems to work well, but is so complicated that it is difficult to intuitively see its basic meaning. It more belongs to process engineering than rheology, but has with time come to be used (and perhaps misused) more widely. The original articles are the most cited in the influential A.I.Ch.E. Journal. Both Duffy and the author think that the method just for this reason deserves an open-minded discussion from different points of view. But when one tries to initiate discussions in the appropriate journals, one is typically met with snubs and sour reactions from "a highly



respected researcher in this field,” giving the impression that a good deal of prestige is hidden here, in turn indicating the need for closer examination.

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