Non-linear viscoelastic behaviour of a suspension of magnetized solid particles under large amplitude oscillatory shear test: A direct numerical simulation

Mohammad R. Hashemi[1], Mehrdad T. Manzari[1,2], and Rouhollah Fatehi[3]

[1] School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran
[2] School of Geosciences, University of Aberdeen, Aberdeen, UK
[3] Department of Mechanical Engineering, Persian Gulf University, Bushehr, Iran

ABSTRACT

A series of test-cases are solved addressing the non-linear rheological behaviour of a periodic array of non-gap-spanning magnetic clusters suspended in a Newtonian fluid. Large amplitude oscillatory shear tests are conducted and the distinct rheology of the system is investigated using Chebyshev coefficients of the elastic and viscous stresses.

INTRODUCTION

Magnetorheological (MR) fluids are made by suspending micron-sized magnetizable solid particles in a non-magnetic liquid. Due to their adjustable rheology, MR fluids are suitable for brand new active control systems. Conventionally, these fluids are modelled using the Bingham constitutive equation that is characterized by a yield stress[1]. However, there is still a limited literature thoroughly investigating MR fluids at the post-yield state.

Upon the application of an external magnetic field, particles are arranged in chain-like clusters that can greatly resist flow[2]. Depending on the intensity of the magnetic field, size, and concentration of the magnetic particles, these chains can span the whole width of the test domain so that a solid-like viscoelastic behaviour is observed. By imposing a rather large shear strain, magnetic chains break most probably from their tips[3], and ultimately a fluid-like behaviour is observed. At this point, magnetic clusters are majorly non-gap-spanning.

This work focuses on investigating the contribution of such non-gap-spanning clusters in the non-linear viscoelastic rheology of an MR fluid by systematically studying their role in determining the effective stress response of a magnetic suspension subject to the large amplitude oscillatory shear (LAOS) test. To achieve this, a direct numerical simulation (DNS) approach[4] is employed.

PHYSICAL MODEL

In this work, nine neutrally buoyant para-magnetic circular cylinders are suspended in a Newtonian fluid and the resulting suspension is confined between two infinitely long oscillating solid walls. Magnetic clusters are formed under the influence of an external magnetic field with a flux density of B_0. The initial configuration is symmetric so that the centre of the magnetic clusters is positioned on the centreline of the channel. The physical model is schematically shown in Fig. 1. The governing equations and the numerical method were presented in an earlier work[4].

In order to study the effective rheology of the system, the (shear) stress response is calculated as

$$\bar{\sigma}_{xy} = \frac{1}{L}\int_{y=0} \sigma_{xy}(x)dx \,, \qquad (1)$$

where σ_{xy} is the spatially averaged stress. For the sake of brevity, the over-bar sign is omitted in the following.

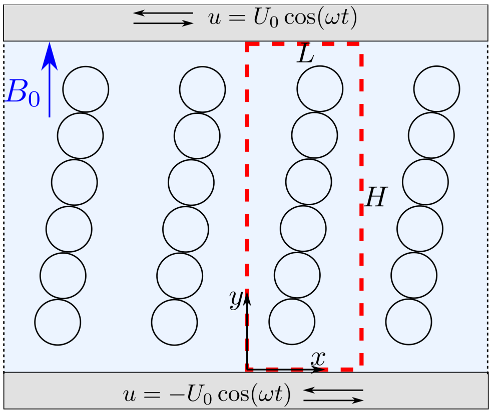

Figure 1. Schematic of the physical model. The computational domain is marked with dashed-lines. Periodic boundary condition is considered at both sides of the domain.

For all cases presented in this article, the ratio of the channel height and periodic width (of the computational domain) to the radius of the solid particles are $H/a = 20$ and $L/a = 8$, respectively. The constant magnetic susceptivity is $\chi = 0.1$, frequency of oscillations is $\omega = 2\pi\,(Rad/s)$, and the particle Reynolds number is set to $Re_p = \rho\dot{\gamma}_0 a^2/\eta_0 = 0.003$, where ρ and η_0 are density and dynamic viscosity of the suspending fluid. $\dot{\gamma}_0 = 2U_0/H$ is the amplitude of the (shear) strain-rate.

RESULTS

Conventionally, under an oscillatory shear strain, $\gamma = \gamma_0 \sin(\omega t)$, the viscoelastic behaviour of the system is determined by calculating the average elastic modulus

$$G' = \frac{\omega}{\pi\gamma_0^2}\oint \sigma_{xy}(t)\gamma(t)\,dt\,, \tag{2}$$

and the dynamic viscosity

$$\eta' = \frac{1}{\pi\omega\gamma_0^2}\oint \sigma_{xy}(t)\dot{\gamma}(t)\,dt\,. \tag{3}$$

In Figs. 2 and 3, G' and η' are shown as functions of B_0, respectively. It is seen that both G' and η' are increasing functions of B_0, while the slope decreases by increasing the strain-amplitude. It is also worth noting that strain-softening and shear thickening behaviours can be observed in Figs. 2 and 3, respectively. That is, for the range of B_0 considered in this work, by increasing the strain-amplitude, G' decreases, while η' increases.

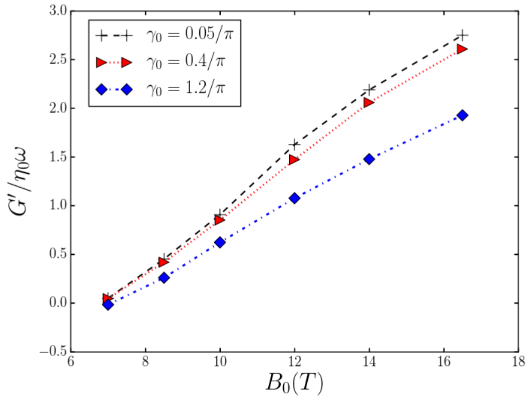

Figure 2. Elastic modulus as a function of B_0 obtained for three strain amplitudes.

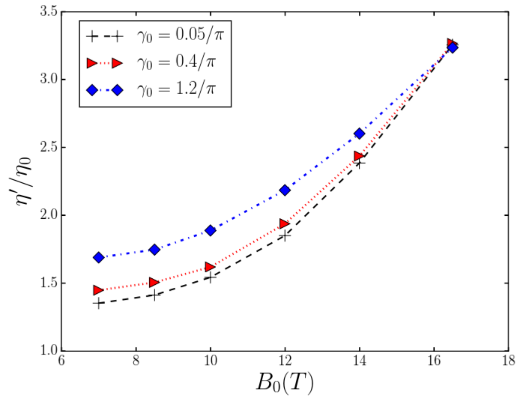

Figure 3. Dynamic viscosity as a function of B_0 obtained for three strain amplitudes.

However, unlike the small amplitude oscillatory shear (SAOS), in a LAOS test, the stress response is not a simple harmonic function. Using Fourier series, it can be written as

$$\sigma_{xy} = \gamma_0 \sum_{n:odd} G_n'(\omega,\gamma_0)\sin(n\omega t) + \omega\eta_n'(\omega,\gamma_0)\cos(n\omega t), \tag{4}$$

where odd and even terms (with respect to the input strain) correspond to the elasticity and viscosity of the system, respectively[5]. Therefore, the average viscoelastic moduli only represent the linear rheology of the system, i.e. $G' = G_1'$ and $\eta' = \eta_1'$, and a more complicated framework shall be used to study the non-linearity of the results obtained in a LAOS test.

Using the properties of the Chebyshev functions, the elastic stress (odd part of the stress response in Eq. 4) can be rewritten as[6]

$$\sigma_{xy}'(\gamma) = \gamma_0 \sum_{n:odd} e_n(\omega,\gamma_0)\mathrm{T}_n(\frac{\gamma}{\gamma_0}), \tag{5}$$

where e_n is the nth elastic Chebyshev coefficient corresponding to the nth harmonic of the stress response and T_n is the nth-order Chebyshev function of the first kind. Similarly, the viscous stress becomes

$$\sigma_{xy}''(\dot\gamma) = \dot\gamma_0 \sum_{n:odd} v_n(\omega,\gamma_0)\mathrm{T}_n(\frac{\dot\gamma}{\dot\gamma_0}), \tag{6}$$

where v_n is the nth viscous Chebyshev coefficient.

Considering that the intensity of the non-linearity of the stress response is mainly determined by the third harmonic, Ewoldt et al.[6] showed that the quality of the non-linear intra-cycle rheology can be determined using sign of the third elastic (e_3) and viscous (v_3) Chebyshev coefficients. That is, in a complete strain-cycle, strain-stiffening and strain-softening are associated with positive and negative e_3, respectively, while shear-thickening and shear-thinning are associated with positive and negative v_3, respectively. These coefficients are shown in Figs. 4 and 5 as functions of B_0. As seen in these figures, e_3 and v_3 vary non-monotonically by increasing the strength of the external magnetic field.

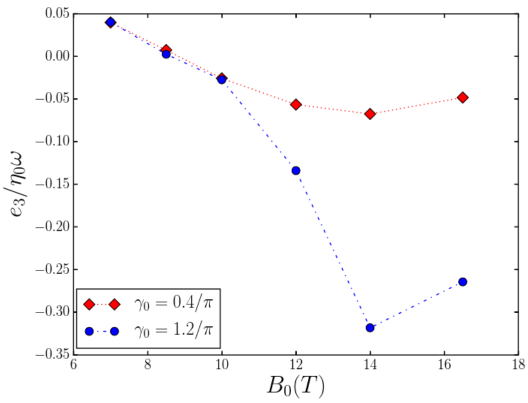

Figure 4. Third elastic Chebyshev coefficient as a function of B_0 obtained for two strain amplitudes.

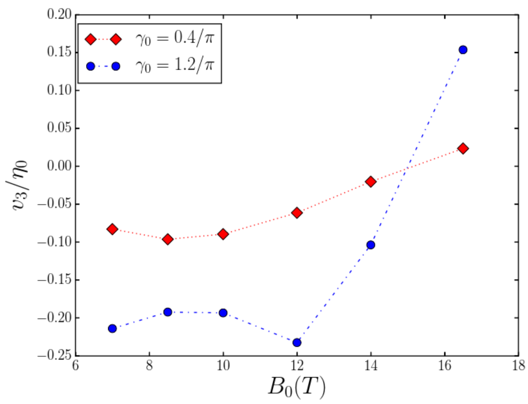

Figure 5. Third viscous Chebyshev coefficient as a function of B_0 obtained for two strain amplitudes.

For the whole range of B_0 considered in this work, e_3 is negative, which means an intra-cycle strain-softening. On the other hand, v_3 is negative for $B_0 < 15\,(T)$ and becomes positive above $15\,(T)$. Therefore, the intra-cycle behaviour is shear-thinning for

$B_0 < 15(T)$ and becomes shear-thickening for $B_0 > 15(T)$.

CONCLUSION

In this work, the stress response of a model suspension of magnetic particles forming non-gap-spanning clusters was investigated by conducting LAOS tests. It was observed that the average elastic modulus is a decreasing function of the strain amplitude. Also, the intra-cycle behaviour of the system was strain-softening. On the other hand, the average dynamic viscosity increased by increasing the strain-amplitude. However, for the same range of intensity of the external magnetic field, the intra-cycle behaviour was shear-thinning. In this sense, as discussed in the literature[7], the non-linearity of the viscous stress response of this system is beyond its intra-cycle behaviour and it can be determined only by conducting further strain-sweep tests.

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