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Methods to Analyze Problems in Coating Flows by Non-Newtonian Fluid Dynamics

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ABSTRACT

In this paper, we discuss on methods to analyze problems in coating flows by adopting non-Newtonian fluid dynamics. We concentrate mainly on two different coating applications: forward roll coating and curtain coating. The material properties of coating colors were defined by analyzing the measurements found in the literature. Because, in coating processes, shear rates usually fall within the shear thinning range, the Power-law model was chosen as a constitutive equation in the simulations with commercial numerical codes.

INTRODUCTION

In paper making, products are almost invariably coated with some coating material to give them the desired properties. The coating materials, often referred to as coating colors or pastes, are usually water based colloidal organic and/or polymer suspensions with high solids content (up to 60% by volume). The materials are shear thinning in nature, and some of them exhibit yield stress behavior at small deformations or even viscoelastic properties.

In this paper, we discuss on methods to analyze problems in coating flows by adopting non-Newtonian fluid dynamics. We concentrate mainly on two different coating applications: forward roll coating, which is one of the most widely used coating applications in paper making processes; and curtain coating.

Forward roll coating

Forward roll coating may be defined as a process where a thin liquid layer of coating color is formed on a continuous moving web between two rotating rolls (see Fig. 1). The process can be divided into two distinct flow types according to the feed condition of the applicator roll, which applies the coating color to the substratum. In a feed condition where the dimensionless volumetric flow rate[1] ($\lambda = q/2H_0U$), applied by the applicator roll, exceeds the natural flow rate capable of passing through the nip, the inlet is said to be flooded ($\lambda > 1$). In a condition where the volumetric flow rate is lower, the inlet is said to be starved ($\lambda < 1$). The flooded inlet flow condition is sometimes referred to as a traditional forward roll coating, and it is the most commonly used roll coating application in the paper making industry. The starved inlet flow condition, also called meniscus roll coating, has certain advantages, which will be discussed later.

Only a very limited area in the vicinity of the nip is of interest, when we examine the flow characteristics in the forward roll coating process, because the flow in the nip area between the two co-rotating rolls determines the thickness and uniformity of the final, coated film. Following four factors determine the flow characteristics: 1) the contact point between the coating material and the upper roll, along which the paper web to be coated is fed into the process; 2) the film splitting meniscus on the downstream side of the nip, where the coating material splits into two distinct films; 3) the inlet condition under which the coating color is applied; and, of course, 4) the rheological properties of the coating colors.

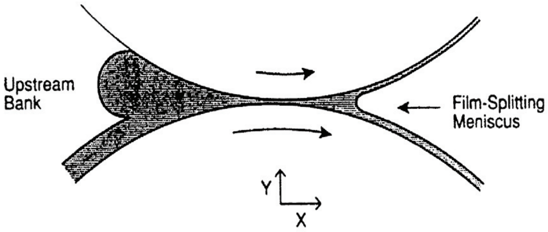

Figure 1. Forward roll coating flow (flooded inlet)[2].

Curtain coating

The principle of the curtain coating process is shown in Fig. 2. The fluid flows from the piping into a manifold from where it is spread evenly to a very narrow slit having the width of the machine wire. After the slit, the coating paste flows on a slide onto the paper. The screw pump creates pulsation, which can be seen as the volume flow rate oscillation in coating paste and finally as the harmonic coating thickness oscillation in paper in the machine-direction, which is a quality defect in paper. The pulsation problem and the resulting basis weight variation have been studied extensively for the base paper but not for the coating[3, 4]. Pulsating and oscillating flows of non-Newtonian fluids have been studied extensively in the literature[5-9].

The equations of fluid transients are used for analyzing the flow when wave propagation is present. This approach can be used in the pressure pulsation analysis in the approach flow system and the headbox of the paper machine in which the fluid is water-fiber mixture[3]. Since the problem of pulsation is similar in the coating process we attempt to conquer this problem using the same method because of its ease of use and very low computational cost. The major difference in these two processes is the importance of modeling of friction. In the former case, it can be shown that friction is not important, whereas in the latter, having a fluid of high viscosity and small dimensions of the duct, modeling of friction is extremely important.

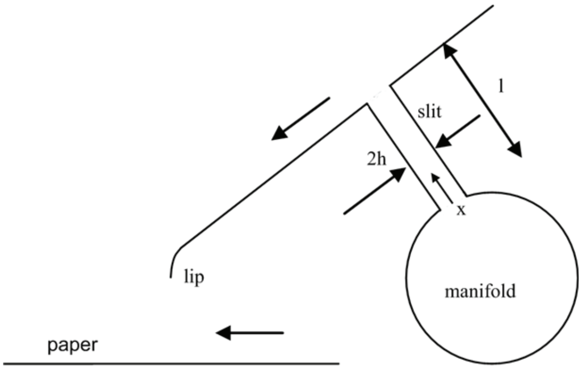

Figure 2. Principle of curtain coating of paper and geometry.

Other coating processes

Reverse roll coating, though quite an old coating process, is still used in many paper mills around the world. It differs from forward roll coating mainly in that the rolls counter-rotate and the coating process takes place almost exclusively on the downstream side of the nip. Reverse roll coating owes its popularity most likely to its high accuracy and widely variable running speeds[13].

Knife coating may be defined as a process where a thin liquid layer of coating color is formed on a continuously moving web under a rigid knife. Color may be applied to the web in several ways: through dipping, roll application, or puddle-feed, and any excess coating is metered by the rigid knife. This is a very simple coating process, but it has the disadvantage of tending to cut the coated substratum, which is highly tensed under the knife. The process shows also problems with stability as the knife is subject to high frequency vibrations in its interaction with the web, resulting sometimes in inaccurate coating[13].

Jet coating is presently one of the most promising new features in paper coating, for it is an "easy" way to coat the paper because it does not require large equipment as some other more traditional coating processes. It is also a process responsive to the challenge of today's modern paper machine with its high web speed[14].

In jet coating, coating color is applied to the web in a free jet with a nozzle speed 0.1 times the speed of the web, which means that the color must accelerate greatly as it hits the web. This is one of the main problems encountered in jet coating, as it causes flow instabilities and hence possible coating defects. Air may also get entrained through the contact point with the web. The problems here are thus basically the same as those in forward roll coating when it runs in the starved mode.

RHEOLOGY OF COATING COLOURS

Coating colors are highly concentrated, usually water based suspensions with a solids content of up to 60% by volume. Synthetic and natural solid particles and polymeric binders all interact with each other, forming complex solutions and exhibiting non-Newtonian behavior at high shear rates and viscoelastic behavior at small deformations. These two distinct behaviors are here discussed separately and then combined to form a unified picture of the rheological behavior of coating colors.

Viscoelastic behavior of coating colors

The rheological behavior of the most common coating colors has been extensively studied in recent decades. At small deformation, they were found to show viscoelastic behavior and it became necessary to characterize the colors showing such a behavior as a function of strain or shear rate. One of the most thorough studies in this regard was carried out by Carreau *et al.*[10], who studied kaolin pigment particles (44 vol. %) in aqueous suspension with and without CMC and SBR latex addition. They used a special kind of rheometer (Bohlin VOR), which mimicked the coating process very closely and enabled both experiments, *i.e.* the constant shear and the small amplitude sinusoidal shear tests, to be performed.

At the moderate shear rates or frequencies ($10^{-3} < \omega < 10^{2}$ 1/s), coating colors showed viscoelastic behavior with the storage modulus showing at least an order of magnitude higher values than the loss modulus[1]. Both moduli showed a plateau at low shear rates ($\omega < 1$ 1/s). The value of the possible yield stress can be extrapolated from the values of the real part of the complex modulus (G^*) when $\omega \rightarrow 0$, or by fitting a rheological model to the measured shear viscosity data. With the Cox-Merz analogy[5], the complex viscosity of matter may be equated with the steady shear viscosity ($\eta^*(\omega = \dot{\gamma}) = \eta(\dot{\gamma})$). With this approximation, the shear viscosity of a pure kaolin clay suspension was analyzed and found to exhibit shear thinning in the whole range of the measured shear rates ($10^{-3} < \omega < 10^{2}$ 1/s)[10]. Furthermore, a hysteresis loop for both shear stress and viscosity was well in evidence[1].

Adding viscosity modifiers (for example CMC particles) had a marked effect on the rheological properties of coating colors. According to Carreau *et al.*[10] even a small amount of CMC (0.25%) increased the value of both the shear viscosity and the storage modulus of a kaolin clay suspension by an order of magnitude, which is much more than expected to be brought about by adding a volumetric concentration. The increase particularly in the storage modulus suggested that the interaction between the kaolin particles changed and that the molecular bindings became much stronger because of the added viscosity modifier. Another addition of CMC (1%) further increased both values in a similar pronounced manner[1].

Adding a synthetic binder (SBR latex) together with a viscosity modifier (CMC) did not markedly affect the shear viscosity of the kaolin suspension[10]. The viscosity values are somewhat higher than without the added SBR but only by the order of the increase in volumetric concentration. Unfortunately, nothing is known about the effect of adding SBR without the presence of CMC on, *e.g.*, the response at low shear rates and therefore the storage modulus of coating colors.

Behavior at high shear rates

In some coating processes, such as jet or blade coating, extremely high ($\dot{\gamma}$ -> 10^6 1/s) shear rates apply to the coating material. In some more traditional coating processes, too, such as roll coating or curtain coating, local shear rates may be relatively high near the contact point with the web. That is why the rheological behavior of coating colors should be determined also at very high shear rates.

One of the best studies on the subject was made by Kurath *et al.*[11], who used a capillary rheometer to measure the high shear rate viscosity of coating colors with natural binders (kaolin clay with hydroxyethylated starch) and with synthetic binders (kaolin clay with calcium carbonate and PVAc or SBR latex) in aqueous suspensions. The solid content of the kaolin clay was about 55% by mass, corresponding to over 40% by volume in suspension. Both types of coating colors exhibited shear thinning behavior at high shear rates, while in a kaolin-starch suspension the behavior was even more pronounced. Fig. 3 shows their results with starch content as a parameter.

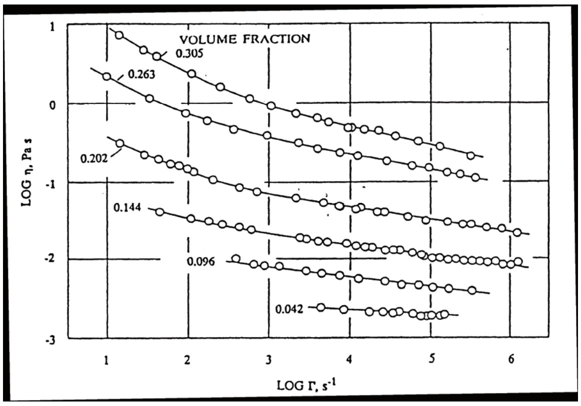

Figure 3. Viscosity of kaolin clay-starch suspension at high shear rates[12].

BASIC THEORY

Coating colors are highly concentrated, usually water based suspensions with a solids content of up to 60% by volume. Synthetic and natural solid particles and polymeric binders all interact with each other, forming complex solutions and exhibiting non-Newtonian behavior at high shear rates and viscoelastic behavior at small deformations. Because there is no multiphase flow model that can take into account all of the above features, it is sometimes the only reasonable way to describe this kind of material by using non-Newtonian fluid models and to use continuum approach. More about the usage of continuum mechanics and non-Newtonian fluid models on the coating processes can be found in the literature[1,3].

When we are dealing with a medium that does not obey the constitutive equation for Newtonian fluid, we must consider it as a non-Newtonian material and find or develop a new model to describe the relation between stress field and deformations or rate of deformations. Therefore, the key issue in the process analysis is to find or develop a suitable constitutive relation to describe the behaviour of the medium under consideration. To do that, we can use non-Newtonian fluid dynamics and generalized Newtonian fluid models.

Non-Newtonian fluid models

In the literature, there is a huge amount of material models describing the material behaviour which does not obey the Newton's law of viscosity, in which the relationship between rate of deformation tensor and stress tensor is linear.

Even though coating colours are found to have viscoelastic features, most of the coating processes work on such high shear rates that the effects of viscoelasticity can be neglected and modelling of coating color behaviour can be done by generalized Newtonian fluid models. For the same reason, also the yield stress behaviour can usually be ignored. Therefore, we used the Power-law model, which can take into account the shear thinning behaviour of coating colours found in the measurements, in our simulations.

$$\tau = K\dot{\gamma}^{n} \qquad (1)$$

In the Eq. 1 n is the Power-law index or shear index, K the consistency factor, and $\dot{\gamma}$ the shear rate, determined by rate of deformation tensor in a usual way[1,5]. Typical behavior of coating colors is shown in Fig 3.

The Power-law model is incorporated in the friction factor used in the equations of fluid transients for curtain coating.

Equations of fluid transients

The equations of fluid transients are the equations of motion and continuity[15]

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial V}{\partial t} + \frac{\xi V|V|^{2}}{2D} = 0 \qquad (2)$$

$$\frac{\partial V}{\partial x} + \frac{1}{a^{2}\rho}\frac{\partial p}{\partial t} = 0 \qquad (3)$$

where ρ is the density of the fluid, x the coordinate in the flow direction, V the flow velocity, ξ the friction factor, D the diameter of the pipe, a the wave speed of the fluid and t the time.

In the frequency domain these equations can be cast in such a form that by knowing a sufficient number of boundary conditions we can calculate the pressure and velocity at any point of the pipe. The volume flow rate at the slit outlet is given by a simple analytic formula[16]

$$Q_{D} = \frac{p_{U}}{\rho g Z_{C}}\left(\sinh \gamma l + \cosh^{2} \gamma l / \sinh \gamma l\right) \qquad (4)$$

where l is the length of the slit p_U the pressure pulsation amplitude at the upstream end of the slit, g the gravitational acceleration Z_C the characteristic impedance and γ the propagation constant, which includes the friction factor and f the frequency. The quasi-steady assumption is used in calculating the friction factor which is valid if the velocity pulsation amplitude is much smaller than the mean flow velocity.

Governing equations for roll coating

The flooded inlet flow condition in the nip area may be analyzed in terms of a one-dimensional flow field by using the lubrication theory, which assumes one dimensional flow in a converging or diverging flow channel. In this case, the Navier-Stokes equations are reduced to only one equation as follows:

$$\frac{\partial p}{\partial x} = \eta\frac{\partial^{2} u}{\partial y^{2}} \qquad (5)$$

In a dimensionless form, and assuming atmospheric pressure far upstream from the nip, the equation can be solved[18] as

$$\frac{\hat{p}}{3\sqrt{2}} = -\frac{\lambda}{4}\sin\theta(\cos\theta)^{3} + \left(1 - \frac{3}{4}\lambda\right)\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{\pi}{4}\right) \qquad (6)$$

where $\hat{p}$ is the dimensionless pressure defined as[18]

$$\hat{p} = \frac{pH_{0}}{\eta\bar{U}}\left(\frac{H_{0}}{R}\right)^{0.5} \qquad (7)$$

Here *R* is the radius of the rolls and the dimensionless *x*-coordinate defined by[19]

$$\tan\theta = \frac{X}{\sqrt{2RH_{0}}} \qquad (8)$$

PROBLEMS IN COATING PROCESSES

In this section we shall discuss the problems in coating flows and methods to solve them.

Forward roll coating

The contact point in the forward roll coating process is very sensitive to any disturbance in the coating flow. The contact point is a singularity point both in the physical and numerical sense, because it is where free surface of the coating color adheres to the upper solid roll. Physically this contact process is very complex and greatly affected by the rotational speed of the upper roll, and the speed ratio between the free surface of the applied coating color and the rotating roll. The stability criteria at the contact point may be written in the form of a force balance for a stationary location of the contact point[1].

Two distinct vortices develop on the downstream side of the nip near the film splitting meniscus, where the fluid particles, following the streamlines, undergo intensive acceleration rates as they rapidly change directions near the interface between air and the coating color. Streamlines in meniscus roll coating are presented in Fig. 4. If the acceleration rates exceed the extensional viscous forces in the coating material, the structure of the material disintegrates, and the fluid particles may shoot out from their original tracks (= streamlines) and even from the whole flow domain through the interface, if the inertial forces exceed the surface forces.

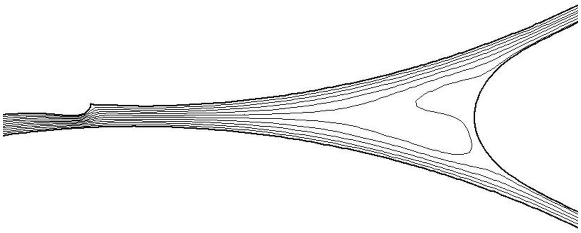

Figure 4. Streamlines of meniscus roll coating flow (starved inlet, $\lambda<1$).

The physical conditions, too, near the contact point may result in coating defects in forward roll coating. If the process is run in the flooded inlet mode, an upstream bank of coating color develops in front of the nip (see Fig. 1), in which case the contact angle, dependent on the surface tension of coating color which usually is quite low ($\sigma < 0.1$ N/m), between the upper roll and the free surface is small ($\beta < 10^o$). The upstream bank itself does not cause coating defects, except through the ageing of the coating material, but it may instigate some primary instability, when it increases to the point where its body forces overcome its surface forces and the bank collapses.

The flooded inlet flow condition in the nip area may be analyzed in terms of a one-dimensional flow field by using the lubrication theory, which assumes one-dimensional flow in a converging or diverging flow channel[1].

When the process is run in the starved feed condition, the speed of the upper roll is usually higher than the speed of the free surface of the coating color. Therefore, the moving web is likely to draw some air along with it into the vicinity of the contact point. The contact angle in this process is high ($\beta \sim 90^o$), and if the surface tension of the coating color is not high enough, the air may be entrained into the coating material at the contact point, in a phenomenon known as skipping. The result is small air bubbles inside the coating color, which prevent coating of the paper web where they attach to it. The bubbles appear on the final, coated paper as small uncoated spots.

A more serious coating defect, called bead break, is also possible under the starved feed condition and happens when the dimensionless flow rate becomes very small ($\lambda << 1$). When the inertial and viscous forces exceed the surface forces, the contact point is sucked through the nip and contact is lost with the upper roll[12]. Should this happen, the whole process must be shut down and restarted. This is why the meniscus roll coating process is feasible only when the surface tension of the coating color is high enough ($\sigma > 10$ N/m) so that the apparent capillary number[1] can become less than unity ($Ca_{ap} < 1$). The apparent viscosity of coating colors is usually relatively high, thus the apparent Reynolds number[1] is also less than unity ($Re_{ap} < 1$). Consequently, the apparent Weber number[1] becomes much less than unity ($We_{ap} << 1$).

Curtain coating

In curtain coating, the feed pump of the curtain coater creates harmonic pressure oscillation and at the same time velocity oscillation. The oscillation propagates through a narrow slit and, therefore, there exists velocity oscillation at the outlet of the slit, and also volume flow rate oscillation, which can be seen as the density or basis weight variation of the coating in the end product.

It has been observed that viscoelastic pulsating flows exhibit resonance behaviour in terms of flow enhancement and great variation in flow rates can occur with changing excitation frequencies. At certain excitation frequencies the flow enhancement increases dramatically just like in the case the vibration amplitude of a beam increases greatly if it is excited at its natural frequency and at a suitable location. The realistic frequency range of the screw pump excitation in the curtain coating process is such that there is very little change in the pulsation amplitude or mean flow obtained in our analysis. If the excitation frequency is increased to several kilo Hertz, we start to notice the resonance behaviour, but as mentioned above, that is not a realistic frequency for this practical case.

SIMULATION AND RESULTS

Both coating processes under concern here were numerically simulated. For the forward roll coating process simulation we used POLYFLOW and for the curtain coating the equations of fluid transients were solved and the results were verified using FLUENT.

Forward roll coating

To simulate forward roll coating flow between two counter-rotating rolls, a two dimensional numerical model was created based on the continuum approach using the finite element method (POLYFLOW, version 3.5.2). A reduced set of Navier-Stokes equations was solved together with the continuity equation.

As all coating processes, roll coating is very sensitive not only in physical but also in numerical terms, because they contain at least one but often two free interacting surfaces. This increases the non-linearity of the processes and contributes to some numerical problems in remeshing of the deforming computational domain. Furthermore, management of the contact point between coating color and the upper roll in a model with two free surfaces is very important, for in numerical sense it is a singularity point.

A free surface boundary condition with a certain surface tension was imposed on the interface between air and the coating color. Three different models were constructed to simulate forward roll coating flow: symmetrical and non-symmetrical models, for half the problem with only one free surface, and one for the whole problem with two free surfaces. The first two were used to simulate the flooded inlet condition, where the dimensionless flow rates $\lambda > 1$, and where the flow field in front of the nip is not very important. The latter was used in the starved inlet condition where the dimensionless flow rate $\lambda < 1$, where the "snake" (*i.e.* back flow through the nip) should appear at low dimensionless flow rates, and where the flow field in front of the nip cannot be neglected[12].

The flooded inlet cases were studied with models consisting of one free surface only. Results of the simulations were compared with those by Coyle *et al.*[18,19] and found to agree well[1] (Fig. 5).

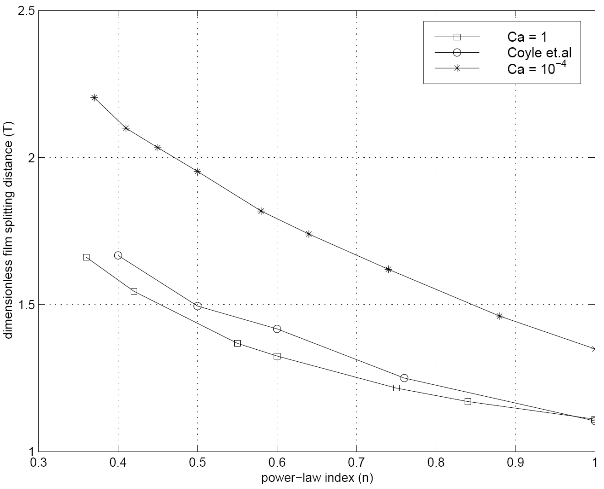

Figure 5. Dimensionless distance of film splitting location as function of shear index. Symmetrical case ($\mathrm{Re}_{ap} = 5 \cdot 10^{-3}$).

Many coating defects in roll coating are due to the instability of nip area and contact point of coating colors to the upper roll. Therefore, we analyzed the simulated pressure distributions in the nip area and acceleration rates of particles in the flow field in vicinity of contact point.

The acceleration rates were high and consisted of two separate peaks with a clear deceleration zone in between them. This can be a very severe condition in contact point for air entrainment and lead to certain coating defect, uncoated spots.

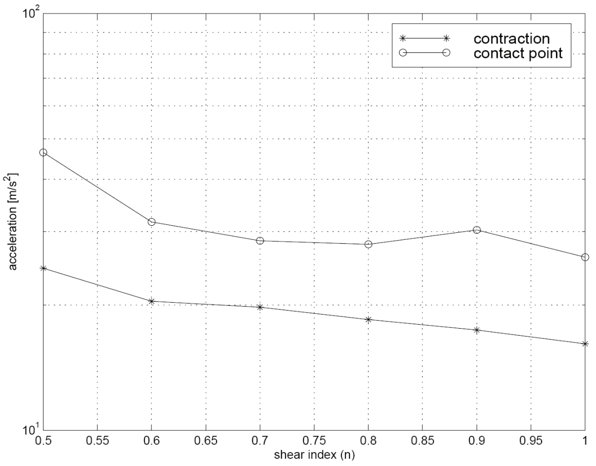

Figure 6. Maximum acceleration rates at contact point and in contraction along streamline.

The other important factor in the nip area is pressure distribution. Even though the starved inlet feeding has economical benefits of saving coating colors, our simulation showed that it is a very unstable coating mode and has a tendency to bead break, the most severe coating defect. That can be seen by studying the simulated pressure distributions in Fig. 7. The relative pressure distribution in starved mode seems to be negative all along the nip area, which indicates the danger of the contact point to be sucked through the nip and to lose the contact with the web.

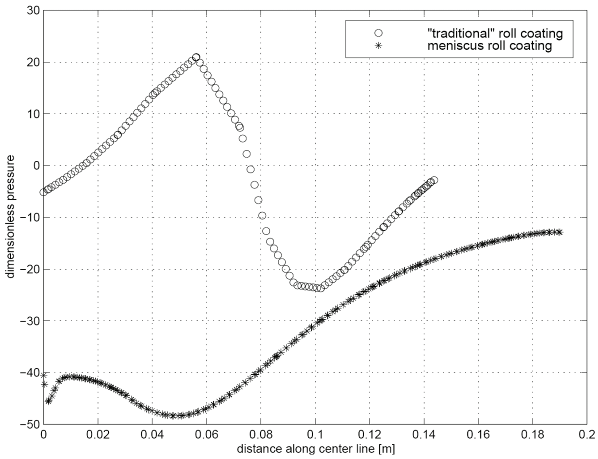

Figure 7. Comparison of meniscus and "traditional" forward roll coating for constant viscosity non-Newtonian fluid.

Curtain coating

The boundary conditions for the problem are the known pressure pulsation amplitude Δp at the upstream end and discharging the flow to the atmosphere corresponds to zero pressure. Furthermore, we know the static pressure difference p_0 which gives the mean velocity in the slit and is approximately 0.2 m/s. The slit length is l = 0.1 m and the height 2h = 0.4 mm. The pulsation frequency is f = 30 Hz. The material parameters corresponding to experiments are $\rho = 1500$ kg/m^3, K = 4.0 and n = 0.35[17].

The velocity pulsation amplitude at the slit outlet as function of pressure pulsation amplitude is illustrated in Fig. 8.

Comparison is made between CFD and fluid transients analyses. We can see that both approaches give similar results at low pulsation amplitudes. This is due to the fact that at high pulsation amplitudes the velocity profile changes significantly and the approach of fluid transients does not take into account this change. The change of velocity profile during one cycle of pulsation is shown in Fig. 9. In practice the pulsation amplitudes are, however, small, and therefore the approach of fluid transients seems to be a promising tool for analyzing pulsation in the curtain coating process.

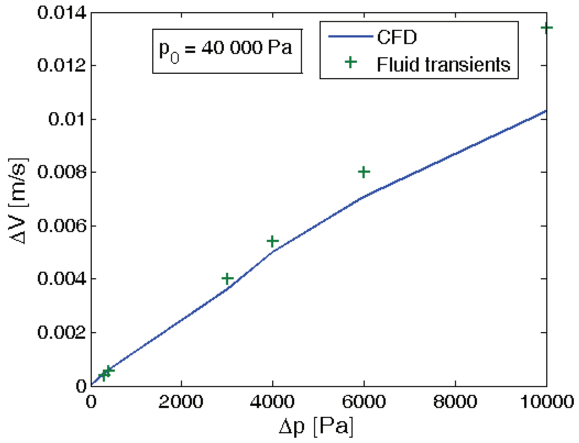

Figure 8. Comparison of velocity oscillation between CFD and fluid transients' solutions for different pulsation amplitudes.

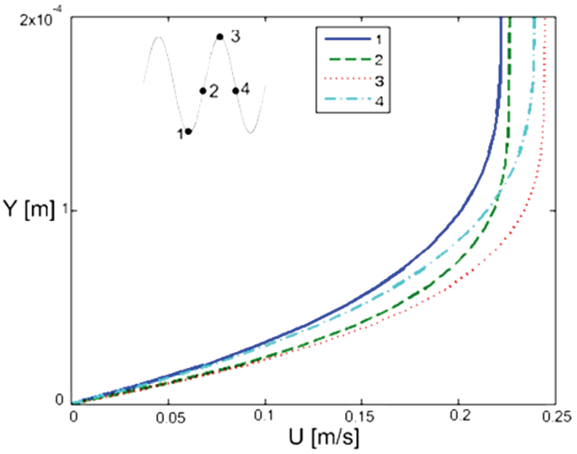

Figure 9. Velocity profile at different instants in pulsating flow in slit.

Simulation results also show that when the pressure pulsation amplitude is increased, the mean flow rate decreases which means that negative flow enhancement occurs. According to the literature this is possible with certain material parameters. A thorough comparison with the literature cannot be made because of the insufficient experimental data for calculating the Deborah and Weissenberg numbers, quantities often used for the characterization of pulsating non-Newtonian fluids. However, in practical applications such as the pulsation problem in curtain coating, these quantities are not needed for predicting the basis weight variation of the coating. The simulated pulsation amplitudes are such that they could correspond to the basis weight variation found in the paper coating, typically approximately 2 % of the mean basis weight[17].

SUMMARY AND DISCUSSION

Numerical simulations of forward roll coating flow yielded some valuable information on the flow situation and stability of the coating process. The flow condition near the contact point between coating color and the upper roll was associated with some coating defects (air entrainment) owing to the material parameters (surface tension) and the inlet feed condition. Flooded feed forward roll coating turned out to be more stable process than starved feed (meniscus) roll coating though also much more color consuming. Since the bead break, the most serious coating defect, is likely to occur with high coating speeds, common in today's paper industry, meniscus roll coating may not be an applicable choice in the paper making process despite its advantages.

The pulsation problem in the curtain coating process was analyzed using the approach of fluid transients and the results were verified using the CFD code FLUENT. It was found that both these approaches give similar results when the pulsation amplitude is small with respect to the mean value, which is the case in reality. Therefore the approach of fluid transient is a suitable tool for analyzing pulsation problems in the curtain coating process instead of the computationally demanding CFD analysis.

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