Modelling rheological cone-plate test conditions

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ABSTRACT

Control with flow conditions is of paramount importance in relation to rheological tests. Special focus is made on the shear rate distribution in the flow volume of cone/plate test geometries and on the shear stress dependence on the central gap.

Small variations in the central gap due to temperature variations may cause significant measurement errors especially for dilatant fluids.

INTRODUCTION

It is often desired to test non-Newtonian fluids at conditions of constant shear rate since the viscosity for such fluids may be highly dependent on the shear rate. Cone/plate test geometries are often chosen for these types of tests since this test geometry approaches the constant shear rate condition in the whole flow domain. The flow conditions in cone/plate geometries have previously been described by Walters[1]. Complex flow conditions can be present if the cone angle is larger then 4º.

However, as will be shown in this study, the shear rate is not constant, and the effects of this can be significant.

The central gap clearance is often set at the beginning of a test when the temperature equals room temperature. If the test temperature is much higher or increases during the test it has been shown[2] that the cone shaft temperature also increases and this may cause the central gap distance to change. The present study assumes that the initial gap is set to 50 µm, and the effects of gap clearance variations are quantified.

BASIC THEORY

Rheological behaviour for fluids with no yield stress is often described by the well known power law equation:

$$\tau = K\gamma^n \qquad (1)$$

where τ is the shear stress and γ is the shear rate. K is called the consistency coefficient and n is the power law index[3]. Differentiation of the equation with respect to shear rate yields an expression for the dynamic viscosity:

$$\eta = \frac{d\tau}{d\dot{\gamma}} = nK\dot{\gamma}^{n-1} \qquad (2)$$

For non-Newtonian fluids, where $n \neq 1$, it is clearly seen that the viscosity is a function of shear rate.

MATERIALS AND METHODS

The geometry studied in this work is a well known cone/plate system (Paar Physica MK22) where the cone angle, α, is 1º and the radius is 25 mm. The default centre distance from the bottom plate during measurements is 0.05 mm. The zero gap setting is normally done during instrument initiation at room temperature.

It is often stated the viscosity measurements in cone/plate systems are performed with a constant shear rate everywhere in the test volume. This is, however, not quite true. Considering the geometry in Fig. 1 we can write the following equation for the angular plate velocity, u, and for the gap, δ:

$$u = \omega r \qquad (3)$$

$$\delta = \delta_0 + r \tan\alpha \qquad (4)$$

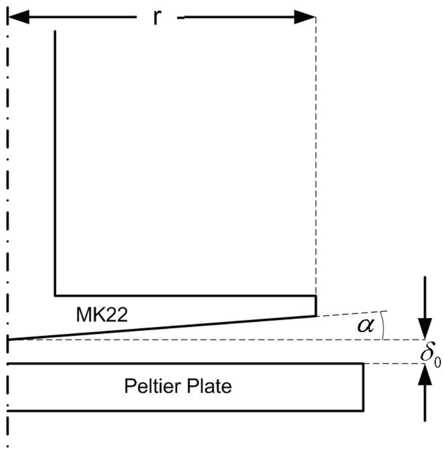

Figure 1: Sketch (not to scale) of cone plate test geometry. Radius is 25 mm, cone angle is 1° and the central gap is 50 µm.

The shear rate can then be expressed by:

$$\gamma = \frac{u}{\delta} = \frac{\omega r}{\delta_0 + r \tan\alpha} \qquad (5)$$

The shear rate calculated from this equation for different values of δ_0 is shown in Fig. 2 with a rotational speed of 1 rad/s.

The torque on the shaft can be expressed as the integral of the shear stress multiplied by the radius over the cone surface, thus:

$$\begin{aligned} I &= \int_0^r \tau 2\pi r dr \\ &= 2\pi \int_0^r K\dot{\gamma}^n r dr \qquad (6) \\ &= 2\pi K \int_0^r \left[\frac{\omega r}{\delta_0 + r \tan\alpha}\right]^n r dr \end{aligned}$$

Analytical solution to this integral is not known, but it is possible to use finite element modelling to calculate the flow domain. However, if we let the gap distance tend to zero, the ideal case with close to constant shear rate, we obtain the simplified approximation:

$$\begin{aligned} I &= 2\pi K \int_0^r \left[\frac{\omega r}{r \tan\alpha}\right]^n r dr \\ &= 2\pi K \int_0^r \left[\frac{\omega}{\tan\alpha}\right]^n r dr \qquad (7) \\ &= \pi K \left(\frac{\omega}{\tan\alpha}\right)^n r^2 \end{aligned}$$

This equation can be solved readily giving the required combinations of *K* and *n* for similar values of torque on the driving shaft, *I*. The radius, *r*, is 25 mm. The solution of Eq. 7 is shown in Fig. 3.

$$\frac{K_1}{K_2} = \left[\frac{\omega}{\tan\alpha}\right]^{\frac{n_2}{n_1}} \qquad (8)$$

Eq. 8 has been used to determine the values of *K* to give the same torque for a pseudo plastic and a dilatant fluid; see Table 1.

Table 1: Values for comparative calculations giving a torque of 0.56 Nm with zero gap clearance.

	K	*n*
Newtonian case	5.00	1.0
Pseudo plastic case	37.85	0.5
Dilatant case	0.087	2.0

The values for the Newtonian case have been arbitrarily chosen.

The shear stress distribution on the cone surface can be determined by combination of Eq. 1 and Eq. 5, thus:

$$\tau = K\left(\frac{\omega r}{\delta_0 + r\tan\alpha}\right)^n \qquad (9)$$

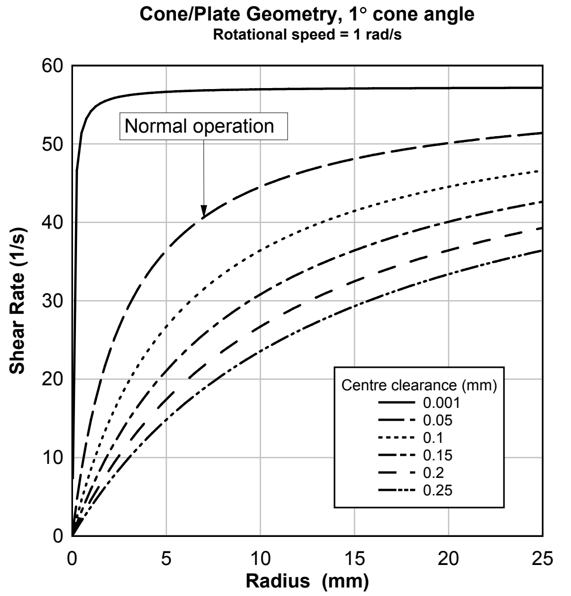

Figure 2: Calculated shear rate distribution in a MK22 cone/plate geometry for different central gap clearances.

FINITE ELEMENT MODELLING

Finite element modelling has previously been used to quantify the lack of temperature control in plate/plate rheometer tests[2]. The calculations were in this study performed with the computer program COMSOL Multiphysics 3.4. The basic geometry used in the calculation was the MK22 set up with a 0.050 mm central gap clearance. All the calculations were made with one rotational speed, $\omega = 1$ rad/s. Both the cone and the bottom plate boundary conditions were specified as non-slip while the fluid to air boundary condition was specified as slip. The cone was specified as a moving wall with tangential velocity, ωr. The main focus was not effects of variation in Reynolds number, Re, being in the laminar region with low values of Re. The number of degrees of freedom solved for was typically 50 000 and the calculation time was approximately 1 minute. A figure of the axis-symmetric flow domain is shown in Fig. 4.

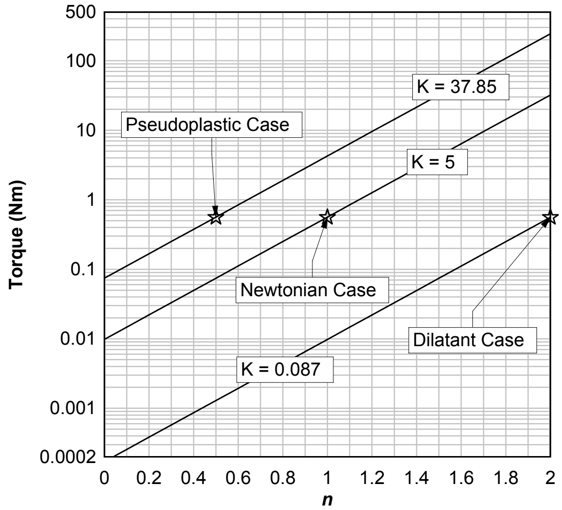

Figure 3: The solution to Eq. 7 and the cases described in for a common torque of 0.56 Nm.

The aim of the finite element modelling was to see if variations in *K* and *n* affected the flow and to investigate how the shear stress distribution on the cone surface was affected by introducing a gap clearance for different types of fluids as specified in Table 1.

RESULTS

The results show that there is no large effect of variations in the consistency coefficient and the power law index on the velocity distribution and therefore only a distribution plot for the Newtonian case is shown in Fig. 5 for the region near the periphery.

Calculations were made for a gap clearance of 0.050 mm and for a gap clearance equal to zero. The results for zero gap show that the shear stress was the same at all radii and the same for pseudoplastic, Newtonian and dilatant flow (Fig. 6).

Figure 4: Normal filled geometry (to scale) of axis-symmetric 2D fluid domain with centre-line on the left. The cone radius is 25 mm and there is a 0.050 mm central gap clearance.

However, when a gap clearance was introduced, the shear stress distributions became those shown in Fig. 6.

The sensitivity to variations in the central gap clearance for the dilatant test fluid is shown in Fig.7.

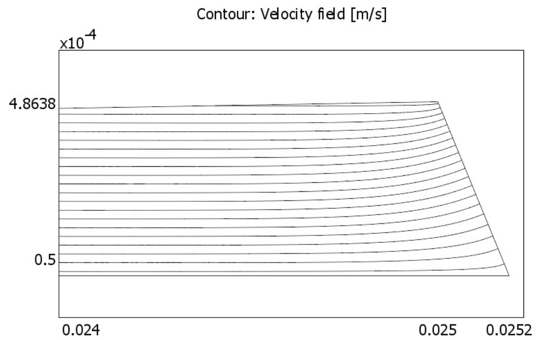

Figure 5: Newtonian iso-velocity lines near the periphery of the cone.

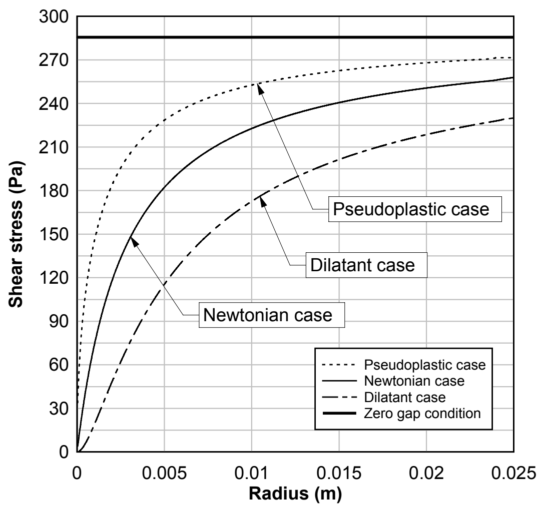

Figure 6: Wall shear stress distribution on the cone as a function of radius for the three basic cases when a 0.050 mm gap is introduced.

DISCUSSION

The length of the rotating shaft is approximately 100 mm, and the part of this that can be affected by changes in temperature is 50 mm, say. The coefficient of linear thermal expansion of stainless steel at 20 ºC is $17.3 \cdot 10^{-6}$ K^{-1}. Calculations show that the changes in the central gap due to a temperature change of 50 K is 43 µm. The central gap clearance must therefore not be set too small to prevent mechanical contact, and the standard value is 50 µm. We see that quite small changes in temperature can give appreciable change in gap size. The main focus of this article is to make an evaluation of the shear rate distributions found in normal cone/plate rheological measurements. It is normally accepted or stated that the shear rate is constant in the measuring volume if a cone/plate geometry is chosen. Calculations show that this is not the case (Fig. 2) and the finite element calculations performed with COMSOL Multiphysics give the same result.

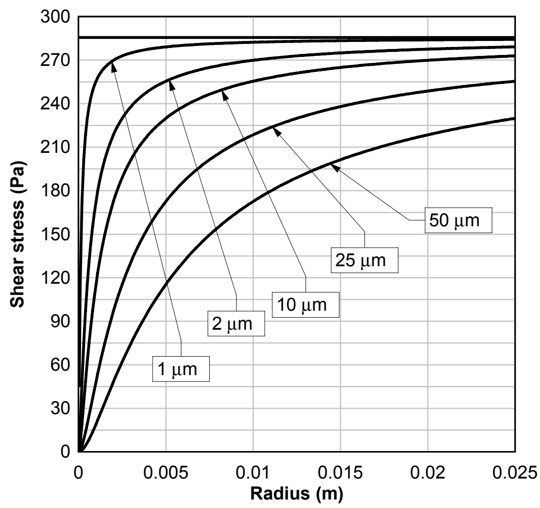

Figure 7: Sensitivity to variation in central gap clearance for the dilatant fluid of Table 1.

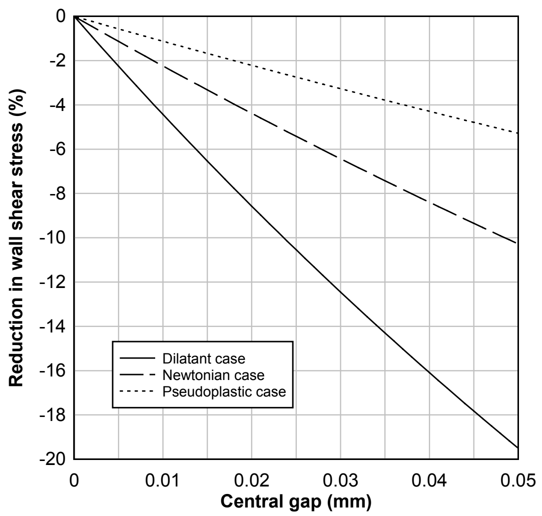

Figure 8: Reduction in shear stress magnitude at r = 25 mm as a function of central gap clearance for the three base cases in Table 1.

We see that the shear rate generally increases with radius for all the cases where the gap is larger than zero. The region of maximum shear rate is located at the cone periphery and the shear rate is zero at the centre.

The results from this study show clearly that cone/plate test geometries do not ensure constant shear rate in the test volume. This would only be true if the central gap clearance was zero, but in practise this gap is significant to make sure that thermal effects on shaft elongation do not cause mechanical contact between the cone and the plate.

When a central gap clearance is introduced, the shear stresses on the cone are generally reduced. This, of course, is a result of the reduced shear rate resulting from increasing the distance between the cone and the bottom plate at all radii. However, it is observed large differences between the pseudoplastic, Newtonian and dilatant fluids. The largest reduction in shear stress is seen with the dilatant fluid.

The sensitivity to changes in the central gap is illustrated in Fig. 7 for a dilatant fluid being most sensitive to changes in the central gap. The chosen reference condition is for zero gap size since the shear stress is the same at all radii in this special case. The effects would, however, be similar if the 50 μm gap was used as the reference position. Even small changes in the gap clearance cause significant change in cone wall shear stress. Thermal expansion effects may cause significant variations in the central gap, so it seems vital that this is controlled in order to minimize measurement errors. Fig. 8 shows the percentage reduction in shear stress level as function of central gap clearance from Eq. 9.

The shear stresses closest to the periphery give the largest contribution to the torque, but it is seen that the shear stresses are also significantly affected in this region when the gap clearance is varied.

CONCLUSIONS

The conclusions from this work can be summarized as follows:

- The shear rate is not constant in normal cone/plate test geometries when a central gap clearance exists. It is zero at the centre and the maximum value is at the periphery.
- The central gap clearance can be affected by a change in temperature of the lower cone shaft causing measurement errors.
- The calculations show that control with the central gap clearance is very important controlling the shear stress distribution on the cone.
- Dilatant fluids are more sensitive to variations in the central gap clearance than Newtonian and pseudoplastic fluids.

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