Constant Force Extension in Polymer Solutions.

Peter Szabo[1], Christian Clasen[2] and Gareth H. McKinley[3]

[1] Department of Chemical and Biochemical Engineering, Technical University of Denmark, Søltofts Plads, Building 229, DK-2800 Kongens Lyngby, Denmark
[2] Department of Chemical Engineering, Katholieke Universiteit Leuven, Willem de Croylaan 46, 3001 Heverlee, Belgium
[3] Department of Mechanical Engineering, Massachussetts Institute of Technology, Cambridge, Massachussetts 02139, USA

ABSTRACT

We consider the stretching of a liquid filament formed by a polymer solution. A liquid bridge is kept between two circular disks. The upper disk is fixed while the lower plate falls due to gravity. This experiment was originally described by Matta & Tytus[1].

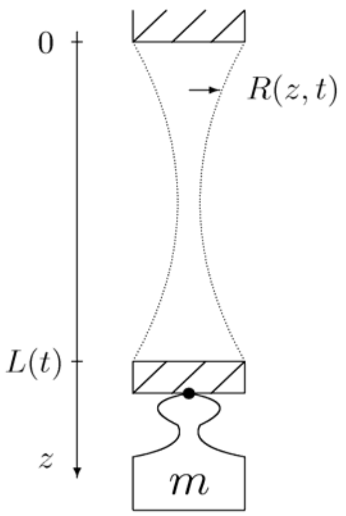

Figure 1. Sketch of filament geometry.

Through variation of the magnitude of the gravitational force the viscoelastic nature of the fluid can be explored. In particular, the maximum extensibility may be quantified from the experiment. This unique characteristic of the experiment is analysed numerically by two alternative techniques that employ either a uniform filament model or a pseudo two-dimensional Lagrangian description of the experiment that use a FENE type model fluid. Based on theoretical considerations we develop an expression for a finite extensibility parameter in terms of quantities that can be extracted from measurement of the time dependent filament diameter. Comparison with experiments is used to evaluate the technique. The developments presented here draws on the work of Szabo et al.[2].

A UNIFORM FILAMENT MODEL

We introduce in the following section a model problem that is relatively simple but captures the key physical features of the falling cylinder experiment. The model allows for simple numerical solution and some analytical insight.

A uniform cylindrical liquid bridge of length L_0 is positioned between a fixed plate and a weight having mass m. At some initial time t_0 the weight is released and starts falling due to gravity. In Fig.1. a sketch illustrates the filament geometry after release of the weight.

For simplicity we do not enforce the no-slip boundary at the endplates and assume the filament remains cylindrical through the thinning process. Experiments for polymer solutions suggest that this could be a good approximation and the plate falls in the same way as a "bungee jumper" attached to a stationary support. However, in contrast to a true bungee jumper who is in free fall until the elastic rope snaps taut, the viscous bungee jumper's fall is slowed by the tensile stress in the elongating fluid thread. Furthermore, we neglect the effects of surface tension in the cylindrical filament and also neglect the effects of fluid inertia. The total force acting on the falling cylinder may then be equated to the gravitational body force minus the resistance in the viscoelastic filament due to viscous and elastic stresses.

The stresses in the polymer solution may be described as a sum of contributions from a Newtonian solvent and the polymeric solute. We obtain the following equation for the length of the filament, L(t):

$$m\, d^2L/dt^2 = mg + [-3\eta_s\dot{\varepsilon} + (\tau_{p,zz}-\tau_{p,rr})]\pi R^2 \quad (1)$$

Here we adopt the notation and sign convention used by Bird et al.[3]. The strain-rate follows directly from the uniform cylindrical fluid filament in which the initial volume is conserved ($R_0^2L_0 = R^2L$) and $\dot{\varepsilon} = (1/L)dL/dt$ for each material element. The polymeric stresses are calculated from a polymer solution model of the FENE-P type:

$$\boldsymbol{\tau}_p = -G\,[\mathbf{A}(1-\mathrm{tr}(\mathbf{A})/b)^{-1} - \mathbf{1}] \quad (2)$$

Here G is the elastic modulus, b is the finite extensibility parameter and **A** is the structure tensor. The evolution of the structure tensor follows the equation

$$\mathbf{A}_{(1)} = -[\mathbf{A}(1-\mathrm{tr}(\mathbf{A})/b)^{-1} - \mathbf{1}]/\lambda \quad (3)$$

where λ is the characteristic time constant (relaxation time).

NON-DIMENSIONAL PARAMETERS

The model problem is conveniently characterised by a set of non-dimensional parameters. A filament length is $\zeta=L/L_0$ wereas we define a time $\tau=t/\sqrt{(R_0/g)}$. These definitions lead to the following non-dimensional parameters:

$$V = (3\pi\eta_s L_0/m)\sqrt{(R_0/g)},$$
$$c = G\lambda/\eta_s \quad \text{and} \quad De = \lambda\sqrt{(g/R_0)} \quad (4)$$

A fourth parameter is the geometric initial aspect ratio $\Lambda=L_0/R_0$.

AN EXAMPLE CALCULATION

In Fig. 2 we show an example calculation of the acceleration of the falling weight for times after release. We observe an initial gravitational acceleration and viscous response followed by a deceleration due to the build up of elastic stresses in the polymer solution. Eventually, gravitational acceleration is approached again as the filament relaxes and thins.

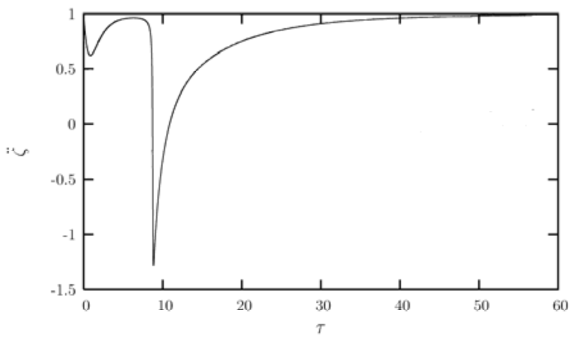

Figure 2. Acceleration vs. time for a FENE-P fluid with c=0.52, b=1000, De=248, V=1 and Λ=1.

REFERENCES

1. Matta, J.E. and Tytus, R.P. (1990), Liquid stretching using a falling cylinder, *J. Non-Newtonian Fluid Mech.*, **35**, 215-229.

2. Szabo, P., Clasen, C. and McKinley, G.H., Constant Force Extensional Rheometry of Polymer Solutions, Submitted to *J. Non-Newtonian Fluid Mech.*, 2010.

3. Bird, R.B., Armstrong, R.C. and Hassager, O., *Dynamics of Polymeric Liquids. Volume 1: Fluid Mechanics.* John Wiley & Sons, New York, 2nd ed., 1987.