Analytical Solutions of the Giesekus and the Phan-Thien and Tanner Models for LAOS

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LAOS is a convenient tool to measure the nonlinear viscoelastic behaviour of viscoelastic materials. Various viscoelastic models have been studied to interpret or predict nonlinear viscoelastic behaviours under LAOS. Nonlinear viscoelastic models consist of linear and nonlinear viscoelastic parameters. Linear viscoelastic parameters are able to be determined by use of several algorithms which have been suggested by many researchers. However, there is no efficient way to determine the nonlinear parameter for LAOS data. Previous researchers determined the nonlinear parameter relying on the other viscoelastic tests such as steady shear test. However, it was found that determined parameters cannot provide qualitative predictions for LAOS data[1,2]. Dynamic regression is suggested by Calin *et al.*[3] to determine the parameter from the LAOS data, but it demands a lot of numerical computations and experimental data.

It is obvious that the analytical solutions of viscoelastic models help to determine nonlinear viscoelastic parameters for LAOS flow as well as understand the nonlinear behaviour of the materials. Analytical solutions for nonlinear viscoelastic models have been studied by use of power series expansion or asymptotic solutions[4,5]. However, there exists inevitable limitation comes from radius of convergence. Consequently, the solutions are valid for restricted region of oscillatory amplitudes.

Various nonlinear viscoelastic models have been studied to investigate the nonlinear behaviour of viscoelastic materials. The Giesekus and the PTT models are popularly used to elucidate the nonlinear flow under LAOS. The constitutive equations of the Giesekus model can be written in terms of extra stress $\mathbf{T}$ as below:

$$\overset{\nabla}{\mathbf{T}}+\frac{1}{\lambda}\mathbf{T}+\frac{\alpha}{\lambda\mathbf{G}}\mathbf{T}\cdot\mathbf{T}=2G\mathbf{D} \tag{1}$$

where ∇ means upper convected derivatives and α is the nonlinear parameter the model. G is modulus and $\mathbf{D}$ is deformation rate tensor. It is known that α of Giesekus model is a constant between 0 and 1. The extra stress of PTT model can be written as

$$\overset{\nabla}{\mathbf{T}}+\frac{1}{\lambda}\exp\left[\frac{\alpha}{G}\mathrm{tr}(\mathbf{T})\right]\mathbf{T}=2G\mathbf{D} \tag{2}$$

It is known that α of PTT model is order of 10^{-2} for solution system and 10^{-1} for polymer melt system[6]. Bae and Cho[7] determined α of Giesekus and PTT models as 0.54 and 0.14 for PEO aqueous solution based on the semianalytical equations of LAOS.

Cho[8] adopted the perturbation method to calculate analytical solutions of the non-separable viscoelastic models. The perturbation method is applied to calculate the analytical solutions of the Giesekus and the PTT models. It is remarkable that it opens a new way to calculate the analytical solution of not only shear stress but also normal stress for LAOS flow.

When $\varepsilon\equiv\alpha/G=0$, the solutions of the Giesekus and the PTT models $\mathbf{T}_0$ are equivalent to that of the upper convected Maxwell (UCM) model.

Because G is sufficiently large for polymer melts or concentrated polymer solutions, $\mathbf{T}_0$ can be remedied by the perturbation method with parameter ε.

$$\mathbf{T} = \mathbf{T}_0 + \varepsilon \mathbf{T}_1 + \varepsilon^2 \mathbf{T}_2 + \cdots. \tag{3}$$

Substitution of lower order solutions provides the first and the second solutions of the Giesekus model as

$$\lambda \overset{\nabla}{\mathbf{T}}_1 + \mathbf{T}_1 = -\mathbf{T}_0^2 \equiv -\mathbf{R}_1^{G} \tag{4}$$

$$\lambda \overset{\nabla}{\mathbf{T}}_2 + \mathbf{T}_2 = -\mathbf{T}_0 \cdot \mathbf{T}_1 - \mathbf{T}_1 \cdot \mathbf{T}_0 \equiv -\mathbf{R}_2^{G}. \tag{5}$$

The constitutive equations of the PTT model provide the analytical solutions as below:

$$\lambda \overset{\nabla}{\mathbf{T}}_1 + \mathbf{T}_1 = -\mathrm{tr}(\mathbf{T}_0)\mathbf{T}_0 \equiv -\mathbf{R}_1^{PTT} \tag{6}$$

$$\begin{aligned} \lambda \overset{\nabla}{\mathbf{T}}_2 + \mathbf{T}_2 &= -\mathrm{tr}(\mathbf{T}_1 + \mathbf{T}_0/2)\mathbf{T}_0 - \mathrm{tr}(\mathbf{T}_0)\mathbf{T}_1 \\ &\equiv -\mathbf{R}_2^{PTT} \end{aligned} \tag{7}$$

The perturbation approximation can be easily calculated by use of integrating transform[8]. It is remarkable that this approach facilitates systematic calculation for analytical solutions of normal stress differences as well as shear stress under LAOS.

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