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Characterization of Non-Newtonian Fluid Models for Wood Fiber Suspensions in Laminar and Turbulent Flows

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ABSTRACT

This research paper deals with the characterization of non-Newtonian fluid models for wood fiber and paper pulp suspensions in laminar and turbulent flows. Those materials are characterized by using the continuum approach, and the flows are expected to obey the conservation laws of continuum mechanics. The study aimed to examine the possibilities of using non-Newtonian fluid models to simulate turbulent flows of the above suspensions. We made rheological measurements to determine the shear viscosity of the suspensions and analyzed the results of other studies in the literature. The studied fluids were shear thinning at moderate shear rates, and concentrated pulp suspensions exhibited some viscoelastic and yield stress behavior at small deformations. At high shear rates and in a turbulent flow field, dilute fiber suspensions had almost water like flow behavior, but fibers had a marked effect on turbulence quantities.

INTRODUCTION

The theories of non-Newtonian fluid dynamics and rheology have for decades been successfully applied, e.g., in polymer processing, but in the papermaking industry it is only now taking its first steps. Attempts have been made to apply non-Newtonian fluid dynamics to laminar flows of pulp suspensions[1, 2, 3, 4, 5], but hardly at all to turbulent flows. Recently, two additional thesis have been written, where the unification of non-Newtonian fluid dynamics and turbulence has been tested in a pressure screen[6] and in a pipe flow[7], and one more[8] for general flow geometries, like refiner, mixing tank, converging channel, and backward facing step flow, with true 3-D nature of turbulence. Also recently, there was a call made by Duffy[20] to find more realistic mechanistic models for pulp suspensions. Here we are trying to answer that call.

In this study, three distinct methods to characterize pulp suspensions' behavior and material parameters are examined. Pipe flow of semi-dilute pulp suspensions ($C < 4\%$) was analyzed as a viscometric flow, because it is a simple shear flow with the velocity gradient only in a pipe radial direction. To be able to determine the shear viscosity of concentrated pulp suspensions (up to 10%), its flow in the vicinity of a rotating disc was analyzed with two different sizes of discs. The results were modified to the form a semi-simple shear flow by subtracting the torque values of the smaller disc from the ones of the larger radius disc. We also re-analyzed the set of rheological measurements made by other authors[5], with an oscillating plate-plate type rheometer, for medium consistency pulp suspensions (C varying from 4 to 8%). They observed the low shear rate characteristics together with linear and non-linear viscoelastic features of the pulp suspensions.

EXPERIMENTS

Non-Newtonian properties of paper pulp and fiber suspensions have been studied for decades, though not reported showing non-Newtonian effects until late 1970s[9, 10, 11]. Pulp suspensions have also been measured rheologically for at least 20 years, mostly pipe flow[3, 4, 7, 10, 14, 19] or flow near a rotating device[3, 5, 6, 12, 13]. These studies have usually focused on some integral quantities, such as pressure drop in pipe flow or the torque of a rotating device, not on any real rheological properties. If done under suitable conditions, however, these measurements can still be of use as rheological measurements.

Rheology is a discipline that observes the behavior of matter and can as such observation be extended to cover matter also in turbulent flow. Though turbulence is always a property of flow, not of fluid, turbulence quantities are always related to the material's behavior because of the scale relation between fluid and flow. As this particular observational approach has not been reported in this form earlier, we may henceforth call it as the study of rheological turbulence. In this section, rheological measurements will be presented of pulp suspensions with a rotating disc in laminar and turbulent flow[8]. Furthermore, two other measurement cases reported earlier, i.e., pipe flow of fiber suspensions[10, 19] and viscoelastic properties of pulp suspensions[5], will also be reviewed.

Rotating disc:

Rheological measurements with a rotating disc for bleached softwood kraft pulp suspensions of 2-10% consistency, mixed with a rotating disc type mixer in a container, were rheologically measured at the laboratory of the Energy and Process Engineering Institute, TUT. The container was designed so as to prevent the walls from affecting the shear stress on the rotating disc and hence its torque resistance. The equipment was run by a 9 kW electromotor, mounted on bearings, and torque resistance was measured on its axle. A transducer controlled the rotational speed of the disc, and a data logger and a personal computer collected data. Two discs of different diameter were used: 280mm and 330mm. The device was described in detail in an earlier thesis[3].

The flow situation near a rotating disc is very intriguing. The set of continuity and momentum equations can be solved analytically but the flow is difficult to analyze, because it is not a simple shear flow and the shear rate on the disc is not constant, but a function of the disc radius. A handy expedient here was to convert the results to a pseudo simple shear flow by subtracting the results of the smaller disc (D=280mm) from those of the larger disc (D=330mm) to learn how much a 25mm disc strip contributed to total torque resistance. The shear rate applied to this strip was assumed constant. The results showed that the 25mm on the outer edge of the disc were responsible for almost half the torque measured. The finding led to another useful approximation: the shear rate on the outer edge of the disc predominates in the process and can be used to determine a pseudo shear rate for the whole rotating disc, a condition which holds especially at low shear rates. These results can also be used to determine yield stress in the creeping flow regime. Moreover, the approximation is useful in shear rate-shear stress analysis and in defining apparent molecular viscosity.

If the tangential velocity component is assumed to dominate near the rotating disc, the shear rate on the disc can be defined with the derivative of the tangential velocity component. In a laminar case, the shear rate on the disc surface may then be determined as a function of the radius by

$$\dot{\gamma}_{R_0} = \left(\frac{r^2 \omega}{\nu_{ap}} \right)^{0.5} \omega g'(\zeta = 0) = 0.6159 \omega \, \mathrm{Re}^{0.5}(r), \tag{1}$$

where g' is the derivative of the dimensionless tangential velocity with a value of 0.6159 at z = 0, according to Schlichting[15]. z is the dimensionless z-coordinate normal to the disc, and Re(r) is the local Reynolds number on the disc and a function of disc radius r.

With this equation, the shear rate on the rim of a rotating disc (r = R_0) can be defined as

$$\dot{\gamma}_{R_0} = 0.6159\,\omega\,\mathrm{Re}^{0.5}\,, \qquad (2)$$

where $\mathrm{Re}=\omega R_0^2/\nu_{ap}$ is the global Reynolds number of the rotating disc.

The above can be used to determine the apparent viscosity of a fluid flowing near a rotating disc in a container. After defining the shear stress on the outer edge of the disc[3], we can write the apparent shear viscosity as

$$\mu_{apR_0}\Big|_{r=R_0} = \frac{(M/2\pi R_0^3)}{\dot{\gamma}_{R_0}}\left[3+\frac{d\{\ln(M/2\pi R_0^3)\}}{d\{\ln(\dot{\gamma}_{R_0})\}}\right]. \qquad (3)$$

We should note, though, that M here is the measured torque of the rotating disc, wetted on one side only and thus half the torque measured in the container (2M)!

The shear stress on the edge of the rotating disc may be defined as[3]

$$\tau_{R_0} = \frac{(2M)_{meas}}{\pi R_0^3}. \qquad (4)$$

With these Eq. (1-4), we can construct either a shear rate-shear stress or a shear rate-apparent viscosity relation to describe the non-Newtonian behavior of paper pulp or fiber suspensions.

Other rheological measurements

The literature offers some other rheological measurements of paper pulp and fiber suspensions, and two of them will be reviewed here, i.e., viscometric measurements of a pipe flow[10, 19] and viscoelastic measurements with a plate-plate rheometer[5].

Pipe flow:

For the Power-law model, then pressure drop in a laminar case can be written as[3, 16, 17]

$$\frac{dp}{dx} = \frac{2K\bar{V}^{n}(3+1/n)^{n}}{R^{(n+1)}}. \qquad (5)$$

If we know the radius of the pipe and can determine the pressure drop and the mean velocity of the flow by measurement, the two material parameters (K and n) can be fitted into the data.

In the turbulent flow regime, the same assumptions can be made as in the Newtonian case. Using the Power-law fluid model, and assuming a viscous sub-layer near the wall, we can write a dimensionless velocity profile (u^+) as a function of dimensionless distance from the wall (y^+)[17]

$$u^+ = \frac{U}{u^*} = \left[\frac{\rho y^n}{k}(u^*)^{(2-n)}\right]^{(1/n)} = \left[y^+\right]^{(1/n)}, (6)$$

where the friction velocity (u*) can be defined with wall shear stress (τ_w) as follows:

$$\left[u^*\right]^2 = \frac{\tau_w}{\rho} = \frac{K}{\rho}\left(\frac{dv_x}{dy}\right). \qquad (7)$$

In the inertial sub-layer, logarithmic law is assumed to describe the velocity profile like in the Newtonian case. By modifying the parameters of the logarithmic law to suit the Power-law fluid model, we can write the law for the friction factor as[17]

$$\frac{1}{\sqrt{f}} = \left(\frac{4.0}{n^{0.75}}\right)\log(\mathrm{Re}_c\, f^{(1-n/2)}) - 0.4/n^{1.2}, \ (8)$$

where the modified Reynolds number is $\mathrm{Re}_c = \rho D^n \bar{V}^{(2-n)}/(8^{(n-1)}K')$, and the modified Power-law consistency is $K' = K\left[(3n+1)/(4n)\right]^n$.

Another way to determine the fiction factor in turbulent flow is to use a relation similar to Blasius's[15] in low-Reynolds-number turbulent flow ($Re < 10^5$). Using 1/8-law for the velocity profile instead of the normal 1/7-law for Newtonian fluids, and fixing the numerical parameter in the equation to fit the measured data, we can write the equation for a Power-law fluid in pipe as

$$4f = 0.0964 / Re_c^{2/9}, \qquad (9)$$

where Re_c is the same modified Reynolds number as defined above.

Based on these definitions, a pressure drop in a Power-law fluid in pipe flow can be determined in the usual fashion as[15]

$$\frac{dp}{dx} = \frac{4f\rho \bar{V}^2}{2D}. \qquad (10)$$

Between these two distinct flow regimes, i.e., fully laminar and turbulent flow, a regime exists where the pressure drop becomes smaller as flow velocity increases. This flow regime is often called the drag reducing flow regime. In paper pulp or fiber suspension flow, the beginning of this regime marks the onset of the plug flow regime, in which a thin fiber-free water film forms between pipe wall and fiber network. Assuming that all the shear stress on the wall applies to the water film, the thickness of the film can be calculated.

If we compare the values of the wall shear stress, calculated from measured data and Eq. (5), assuming the driving pressure force to equal the wall shear stress, we can construct a mathematical model for the slip condition of pulp suspension flow in a pipe. The slip on the wall may be considered a measure of wall shear stress transferred to the fiber network by the thin water film. This theory and its ability to describe the phenomenon have been elaborated more in the earlier thesis[3].

The measured data used here was adopted from an article by Hämström et al.[10], which examined the pipe flow of dilute fiber suspensions (0.67%, 1.27%, 2.47% and 3.41%) of bleached softwood kraft pulp. These measurements were re-analyzed because of their good documentation and convenience, and they agreed well with the other similar measurements in the literature[9, 11, 19]. However, we should note that because different pulp and fiber suspensions show distinct material behavior, they cannot be fully compared. Therefore, measurements with fiber suspension made of a certain wood raw material should be compared only with measurements of pulp of the same wood species and manufacturing methods, because those factors determine how the fiber suspension behaves. This is actually why rheological measurements of different pulp and fiber suspensions are so important, and why these measurements should be made of that very same suspension running in the process under consideration.

Viscoelastic measurements:

Because of wood fibers' exceptional properties to absorb surrounding water and swell, the volumetric concentration of pulp suspensions can be many times the percentage of their mass concentration. Such highly concentrated suspensions with rod-like, slender particles (aspect ratio ~ 1/100) capable of bending and forming agglomerates, i.e., flocs, and coherent networks are often viscoelastic. We could not, however, study viscoelastic properties with our rotating disc type device or in a pipe flow, because the study calls for equipment that can measure the dynamic response of the material.

Swerin et al.[5] studied the linear and non-linear viscoelastic properties of bleached softwood kraft pulp suspensions ($3\% < C < 8\%$) with a plate-plate rheometer. They took measurements at straining frequencies of 0.001-0.3 Hz and at edge strains of 0.002-

0.3, which corresponded to straining amplitudes of 0.06-7% of plate separation. This separation was several times the fiber length to avoid bridging between the plates by fibers. They found that pulp suspensions do have viscoelastic properties in both linear and non-linear regions, which is not surprising, as Damani[1] earlier reported similar results.

The above measurements showed that both the storage and loss modulus of paper pulp suspensions are strong functions of fiber consistency at a given edge strain or straining frequency. Both of these material properties were also strong functions of edge strain at a given consistency or straining frequency, whereas edge strain and fiber consistency had a marked effect on the shape and values of the loss modulus. The value of the storage modulus was a strong function of fiber consistency. However, the slope of the curve for each consistency was almost the same, suggesting that the critical shear stress of a pulp suspension is a function of fiber consistency but the critical strain is not. Other investigators using a rotational viscometric device[18] had also suggested the same.

RESULTS

This section discusses and evaluates the results of the experiments in this study. The results of the rheological experiments were analyzed in terms of continuum mechanics to determine the behavior of the studied materials, i.e., paper pulp and fiber suspensions.

Pipe flow measurements:

We analyzed the measurements by Hemström et al.[10] of bleached softwood kraft pulp suspensions, with consistencies varying from 0.67% to 3.41%, flowing in a straight pipe. The results agree well with other results in the literature[9, 11, 19]. The pressure drop in the pulp suspension flow was a function of fiber consistency at a given flow rate and was very accurately predicted by the Power-law model with Eq. (5) in the laminar flow regime. The curve fitting of each suspension has already been reported in the earlier work[8]. We may conclude that, in the laminar flow region, the Power-law model describes comprehensively the rheological behavior of paper pulp suspensions flowing in a straight pipe, and that more developed generalized Newtonian fluid models are not needed. The Power-law material parameters in Eq. (5) for each suspension analyzed are shown in Table 1.

Table 1. Material parameters for Power-law model[8].

C [%]	K [Pa s^n]	n [-]
0.67	0.44	0.44
1.27	1.20	0.44
2.47	3.80	0.40
3.41	7.99	0.40

The situation is different in the plug flow regime, i.e., the drag-reducing regime, where we do need a model to describe the formation and development of a water film between fiber network and pipe wall. In the earlier section, the thickness of the water film was defined, and it was used in analyzing the pipe flow measurements. The results are shown in Fig. 1. With definition of the slip factor[3] between pipe wall and fiber network can be used to describe the effects of a developing water film. Furthermore, in numerical simulation, the slip factor can be used to introduce a slip

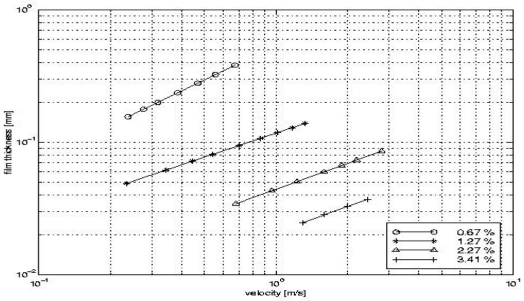

Figure1. The water film thickness[8].

boundary condition on the pipe wall and to obviate explicit modeling of the thin water film and an indefinite boundary between fiber network and water.

In the turbulent flow regime, paper pulp and fiber suspensions seem to behave much like water in a large scale. But as the apparent viscosity of the material is not constant, the shear and strain rate fields tend to modify the molecular viscosity field and thereby the effective viscosity field and, further on, the whole main flow field. This can be seen in measured values[11], because the pressure drop in fiber suspension flow is less than in pure water flow at the same flow rate, even though the apparent viscosity of the suspension is higher than that of water. The pressure drop reduction here can also be a consequence of some wall slippage in turbulent flow regime, but more likely a product of fibers dampening the turbulent velocity fluctuations. Fig. 2 shows the pressure drop values.

Measurements in a mixing tank:

Some principles of bleached softwood kraft pulp suspensions' analysis, flowing in a mixing tank near a rotating disc, were introduced in the earlier section. Torque measurements were reported in the earlier work[3]. Fig. 3. shows the torque measurements as shear rate-shear stress data and Fig. 4. as shear rate-apparent viscosity data for the 25mm strip of the disc's outer rim.

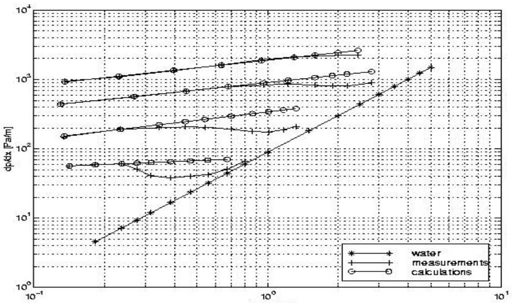

Figure 2. Measured and fitted pressure drop values[8].

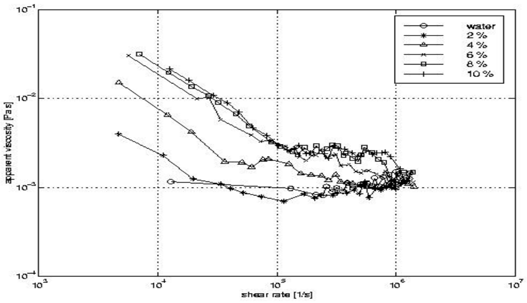

Figure 3. Shear rate vs. shear stress curves[8].

The shear thinning region in the paper pulp suspensions is observable in the measured data at shear rates $\dot{\gamma} < 10^5$ 1/s. Even more pronounced it is in the smaller disc measurements. Yield stress behavior is also apparent at small shear rates, as all shear stress curves level out as the shear rate approaches zero. Furthermore, the abrupt change shown clearly in the shear stress curve between 10^4 to 10^5 1/s indicates the start of a second Newtonian plateau. This behavior is even more evident in the shear rate-apparent viscosity curves in Fig. 4, where the apparent viscosity of the pulp suspensions seems to level out or even increases as the shear rate further increases. Accordingly, some authors call the change point the fluidization point[11], as it marks the point where a paper pulp suspension begins to behave like a pure viscous fluid. Yet the definition is not clear.

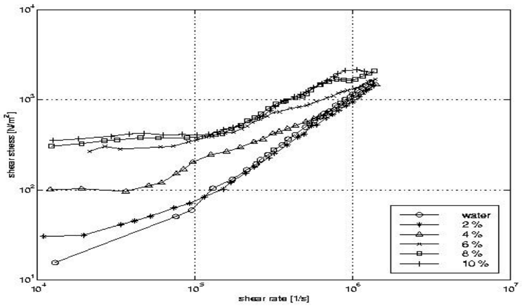

Figure 4. Shear rate vs. apparent viscosity curves[8].

By fluidization point other authors[12, 13] mean the point where the flow resistance curves of water and pulp suspensions intersect and where pulp suspensions turn fully turbulent. In this study, even though the former is important in terms of power consumption, our interest lies in the rheological properties of pulp suspensions and the interaction between suspension flow and turbulence.

According to Figs. 3 and 4, only minor drag-reducing effect of the fiber free water film between disc wall and fiber network can be observed in the laminar flow regime. This is because the actual shear rate on the disc is a function of the disc radius and because, at the same rotational speed, pulp suspensions undergo a variety of shear conditions from high shear on the rim to low shear in the rotational center. That is why constant shear rate data was created, by taking into account only a small strip of the disc rim. Analysis of the measurements was further complicated by the measurement sets not being standardized. Hence the discs' rotational speeds were not necessarily the same at a given input voltage from the transducer, and it was necessary to do some averaging to fit the data for the discs by fitting some polynomials in the measured data and then subtracting them from each other.

The shear rate-shear stress data, obtained via subtraction, reflects behavior similar to that observed in the pipe flow measurements. In higher consistency pulps ($C > 4\%$), the shear stress on the surface of the disc momentarily somewhat decreases as the shear rate increases, indicating the formation of a fiber-free water film on the outer edge of the disc. Because of the slip, the disc "feels" that the material has a lower apparent viscosity than it really does. This phenomenon might, in fact, give us an opportunity to use rotating disc type instrumentation to approximate the wall shear stress and thereby pressure loss in pulp suspension flow in a pipe, where the surface area is the same as that on the disc strip. This would be helpful also in the sense that the costs of rotational disc instrumentation are far lower than those of constructing a whole pipeline.

Table 2. Yield stress values of measurements[8].

C [%]	τ_y [N/m^2] (strip)	τ_y [N/m^2] (Swerin)	τ_y [N/m^2] (Bennington)
2	26.83	6.73	48.99
4	106.7	40.80	245.4
6	244.2	117.1	619.8
8	340.5	247.4	1204
10	409.9	441.9	2017

Yield stress values for both discs can be determined by extrapolating the shear stress curves to a zero shear rate. In Table 2, extrapolation results are compared with those by other researchers who have suggested that the yield stress of a paper pulp suspension should have a power-law relation to fiber consistency as follows:

$$\tau_y = AC^B , \qquad (11)$$

where A and B are parameters depending on the material properties of the pulp suspension under consideration. According to Swerin et al.[5], A = 1.11 and B = 2.6, and according to Bennington et al.[18], A = 9.98 and B = 2.31. The variation in the data is striking. In fact, the values of constant A between 1.18 and 24.5 and those of exponent B between 1.26 and 3.02 have been reported[14]. The scattering in the data may result from different measurement techniques, but more likely, from different pulp suspensions. Our results are shown graphically in Fig. 5 and compared with data available in the literature.

In the turbulent flow regime, the molecular viscosity of the fluid is not a relevant measure, because turbulence adds diffusion and dissipation resulting from velocity fluctuations. These so-called Reynolds stresses act like molecular stresses and produce additional virtual turbulent

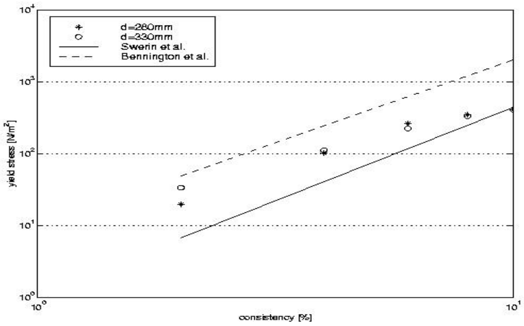

Figure 5. Yield stress values of measurements[8].

viscosity. If measurements are analyzed in the turbulent flow regime, the response, as the net effect of these two types of resistances, is often called effective viscosity. Therefore, to analyze material properties in turbulent flow regime, we should use the equations of turbulent flow.

Viscoelastic measurements:

Since we could not measure viscoelastic properties of the paper pulp suspensions, we adopted and analyzed rheological measurements by other researchers. Some of these measurements, characterizing viscoelastic properties of pulp suspensions, were reported earlier in[3, 5], but a few additions and further specifications may be made here. The analysis was made by using the continuum approach and by following the linear theory of viscoelasticity.

At low frequencies (ω < 1 Hz) and at small edge strains ($\dot{\gamma}$ < 0.01), both, the storage modulus (G') and the loss modulus (G''), approach constant values for a certain consistency pulp suspension. The values of both moduli are shown in Fig. 6 with values of the complex modulus (G*) and the first normal stress, which may be approximated from the data by means of the following relation:

$$\left|G^*\right| = (G'^2 + G''^2)^{0.5} \ , \qquad (12)$$

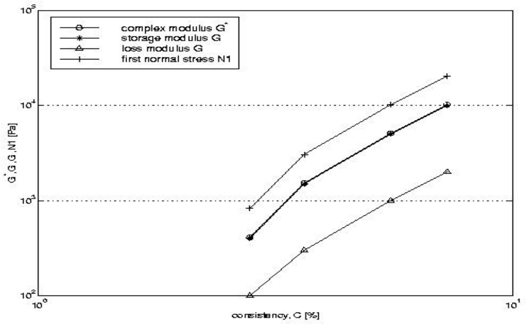

Figure 6. Values of moduli and normal stress[8].

$$\frac{N_1}{2\dot{\gamma}^2} \cong \frac{G'}{\omega^2} \ . \qquad (13)$$

Shear stress and complex viscosity (μ*) can be determined from the measurements as reported earlier[3]. By using the Cox-Merz rule[16] at low shear rates, we can determine the dynamic shear viscosity (μ) of the material by means of complex viscosity as follows:

$$\left|\mu^*(\omega)\right| = \mu(\dot{\gamma}) \ . \qquad (14)$$

With these approximations, the shear viscosity of pulp suspensions seems to exhibit shear thinning even at very low shear rates ($\dot{\gamma}$ ~ 0.01 1/s), and the shear index of different pulp suspensions appears approximately the same for all when C > 2%. This turned out to be the case also at

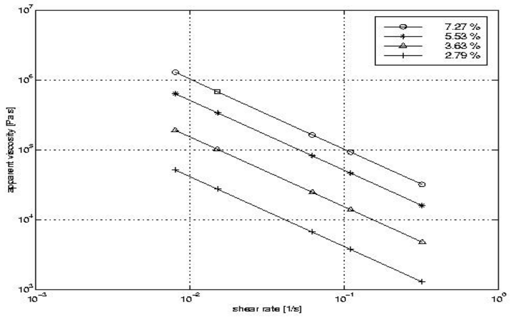

Figure 7. Values of apparent viscosity[8].

high shear rates ($\dot{\gamma} > 10^3$ 1/s), which again suggests that parameters such as fiber length, aspect ratio, and fiber orientation can be more crucial than fiber concentration to determine the rheological behavior of paper pulp suspensions. The results are shown in Fig. 7.

CONCLUSIONS AND DISCUSSION

This study examined the behavior of paper pulp and wood fiber suspension flows in the papermaking industry with focus on experiments. Both the above materials are concentrated suspensions, and their response to a shear stress or shear rate field was analyzed in terms of rheology. The behavior of pulp and fiber suspensions was observed in three different cases: in pipe flow, in rotational flow in a mixing tank near a rotating disc, and in a rheometer between two discs.

In pipe flow, pressure drop measurements were used as rheological measurements. Constitutive equations were obtained to describe the flow behavior of fiber suspensions in pipes with appropriate boundary conditions. The Power-law fluid model with a slip boundary condition well described the behavior of pulp suspensions in laminar pipe flow; thus other, more complicated, constitutive equations were not needed. The same fluid models were used also in turbulent flow calculation, and a Blasius equation type correlation was determined for the friction factor of turbulent flow. Material parameters determined in laminar flow analysis applied also to the turbulent flow regime.

Near the rotating disc, more advanced models were needed to study rotational flow in the mixing tank. Because pulps showed marked yield stress behavior, the Bingham plastic and Herschel-Bulkley models turned out to be the most reliable means to predict the velocity field and torque resistance of the disc in the laminar case, with POLYFLOW. However, the Power-law model failed, by more than 100%, to predict the measured torque resistance. Despite of that, we used the Power-law model to describe the behavior of pulp suspensions in turbulent situation with the same parameters obtained earlier by measurements. Clearly, more enriched fluid models would have been needed for more accurate prediction of turbulent flow, with FLUENT.

In conclusion, continuum mechanics and non-Newtonian fluid dynamics combined with turbulence theory turned out to be a suitable theoretical framework to simulate flows in the papermaking processes. Though we have to keep in mind the limitations of continuum models and rheological measurements, numerical simulation does offer obvious advantages in flow conditions such as these, where accurate measurements are tedious and time-consuming and sometimes even impossible. Simulation of concentrated suspensions in turbulent flow is practically impossible by means other than homogenous models, such as non-Newtonian fluid models. But together with proper turbulence models and wall treatment schemes, these fluid models yield quite reliable results compared to flow measurements. Therefore, this combinatory approach of non-Newtonian fluid dynamics and turbulence theories can be considered successful and one of the most interesting future fields of modern flow simulation.

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