# Rheology: from process to simulation

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#### ABSTRACT

Numerical simulation of polymer processing, or computational rheology, is a challenging conversion of complex material properties and operating conditions into mathematical models and numerical solvers. In this contribution, we present a quick overview of a reasonable methodology with appropriate suggestions for performing a process simulation with chances of success.

#### INTRODUCTION

Polymer processing has become a technologically sophisticated and commercially important activity. This is especially true when considering the yearly production of polymers and the various applications where polymers are involved. End-use products can be as diverse as packaging or mulch films, fibres, bottles, tubes and pipes, etc. A general trend is an increasing demand for tailor-made grades and consumer goods.

The development of a new product often requires a trial-and-error procedure in order technological, fulfil various to the environmental and economical constraints. Just-in-time delivery may require shortening development timelines. The development of a new polymer grade for a given application also needs a validation phase. Those are illustrative and typical situations where a numerical simulation can be considered as a preliminary virtual experiment endowed with several advantages.

In the present context, we focus on polymer melts, and we are therefore interested with properties and behaviours during processing. A careful inspection of polymer melts reveals the development of interesting and often anomalous effects<sup>1</sup>. Experimental observations such as shear thinning, strain hardening, extrudate swelling, rod climbing, slow secondary motions, etc. are attributes of viscoelasticity. They are actually the fingerprint of specific macromolecular architecture and topology<sup>2</sup>. These viscoelastic effects develop with an intensity that depends on the process and operating conditions. Shear thinning and swelling prevail in extrusion, while elongational effects dominate in fibre spinning or film casting<sup>3,4,5</sup>.

The purpose of rheology is multi-folded. Without being exhaustive, it can provide a general framework for the analysis and description of macromolecules. It can constitute a bridge between macromolecular topology and processability. It serves also as a bridge between process and simulation. Indeed, the knowledge of basic properties selection a suitable enables the of mathematical model that will subsequently be used for providing predictions on a given process.

As can be seen, the topic is very broad, and still today it offers enough experimentation ground. Our purpose in the present contribution is to provide a few guidelines for using rheology as a bridge between process and simulation. In the next two sections, we briefly overview usual rheological measurements for polymer melts, with some of their attributes, and recall some empirical rules. The following two sections are dedicated to constitutive modelling and numerical simulation, while a few guidelines for a relevant selection of material parameters are presented in a subsequent section. The last section is dedicated to some selected numerical simulations accompanied by experimental validations.

#### VISCOMETRIC AND RHEOMETRIC MEASUREMENTS

A convenient way for characterising the processability of a polymer melt consists of performing rheological measurements. Here, the melt index is probably the simplest The procedure is well measurement. documented, and the result is usually a single point data. Such a data allows estimating global processing quantities, like the required power consumption for an extruder, etc. However, it does not necessarily allow discriminating between two different melt grades. More intimate rheological measurements are therefore required.

The technology offers a broad range of rheological measurements<sup>6,7</sup>. It is reasonable to consider that the most common rheological data are successively the linear properties, the non-linear shear viscosity and the elongational viscosity. Of course, we understand that an extensive range of measurements can be costly and is not always achievable. Also, it is not always necessary to perform all measurements: here it can be relevant to consider the kinematics contribution that dominates in a process.

Commonly, linear properties are measured in oscillatory regime under small deformation amplitude. A frequency sweep allows the determination of both storage and loss moduli G' and G'' vs. frequency  $\omega$ .

For most polymer melts, the measurement of these linear properties is feasible. In particular. using the time-temperature equivalence allows broadening the range of frequencies, provided that the sensitivity with respect to temperature is sufficiently pronounced. The most typical device for such measurements is the cone-plate However, other oscillatory rheometer. techniques can be considered, such as oscillatory squeeze flow<sup>8</sup>, while a closed device may be required for the measurement of oscillatory properties of rubber<sup>9</sup>.

When feasible, the steady shear viscosity  $\eta_s$  is measured, e.g. in capillary rheometry; it provides shear viscosity vs. apparent shear rate or possibly vs. actual shear rate  $\dot{\gamma}$ . Capillary rheometry also enables data acquisition at relatively high shear rates, as well as on swelling. It is also interesting to note that the multi-pass rheometer allows the measurement of viscosity data at various pressure levels<sup>10</sup>. Capillary measurements reveal the amount of melt shear thinning. As most processes involve shear, such a property is usually of interest.

Finally, transient uniaxial elongational viscosity  $\eta_e^+$  can be of interest<sup>11,12</sup>. This is especially true for processes involving a kinematics dominated by extension effects, such as fibre spinning or film casting. Up to recent years, a device based on rotating clamps was probably the most advanced tool for the measurement of elongational viscosity at constant strain rate  $\dot{\epsilon}^{13}$ . Recent progresses have led to the development of a new apparatus that takes advantage of rotating device technology<sup>14,15</sup>.

Of course, the above paragraphs do not provide a complete list of all possible measurement devices; it would also be beyond the scope of this contribution. Other techniques exist, such as the filament stretching<sup>16</sup>, the Rheotens<sup>17</sup>, the cross-slot flow<sup>18</sup>, the opposed nozzles<sup>19</sup>, etc. Other mathematical frameworks do also exist, such as large amplitude oscillatory shear, e.g. combined to Fourier transform analysis, for macromolecular and topological investigations<sup>20-22</sup>.

# EMPIRICAL RULES AND OBSERVATIONS

As already stated, it is not always necessary to perform all the above measurements. Either this is not required by investigation, the process under or alternatively, possible missing information can (cheaply) be obtained through the use of an empirical rule. Here, the Cox-Merz rule is probably the best known<sup>23</sup>: it provides data on shear viscosity  $\eta_s(\dot{\gamma})$  from linear oscillatory data G' and G'', as follows:

$$\eta_{s}(\dot{\gamma}) = \frac{\sqrt{G'^{2} + G''^{2}}}{\omega} \mid_{\omega = \dot{\gamma}}$$
(1)

As this is an empirical rule, there is no theoretical background; and usage should preferably be restricted to materials for which the validity is generally accepted. Another Cox-Merz rule establishes a relationship between the storage modulus G' and the first normal stress difference  $N_1(\dot{\gamma})$ , and can be given by:

$$N_1(\dot{\gamma}) = 2G'(\omega) |_{\omega = \dot{\gamma}}$$
(2)

This validity of this rule is admitted for low frequencies. Some developments have led to more general formulae, such as:

$$N_{1}(\dot{\gamma}) = 2G' \left[ 1 + \left(\frac{G'}{G''}\right)^{2} \right]_{|\omega=\dot{\gamma}}^{0.7}$$
(3)

which has been validated for several polyethylene melts<sup>24</sup>.

This quick overview of empirical rules would certainly be incomplete without the Gleissle mirror relationships<sup>25</sup>. They express a geometrical symmetry between steady and transient properties. The first Gleissle mirror relationship relates the transient and steady shear viscosities:

$$\eta_s(\dot{\gamma}) = \eta_s^+(t) \mid_{t=1/\dot{\gamma}} \tag{4}$$

As can be seen, measurements of the transient shear viscosity at low shear rates could be sufficient for determining the steady shear viscosity. Let us remind that this is an empirical rule, and that its usage for a given polymer class is subjected to appropriate validations and general acceptance. Other mirror relationships are dedicated to the evaluation of first and second normal stress differences.

Next to those quite useful empirical rules, which are quantified in mathematical terms, there are qualitative empirical rules, which can also be useful. An interesting empirical observation states that the bilogarithmic plot of the first normal stress difference vs. shear stress has a slope near  $2^{6,7}$ . Combining this with both Cox-Merz rules could possibly provide extended data on the first normal stress difference, only on the basis of linear measurements. Indeed, shear stress is extracted from Eq. 1, early development of first normal stress difference is extracted from Eq. 2; while the above bilogarithmic plot extends this data.

Another qualitative empirical observation transient concerns the elongational viscosity. Today's technology allows the measurement of this property for strain rates that are often below those encountered in processing. However, when available, experimental data indicate that the transient elongational viscosity develops along the linear regime and departs from it at a Hencky strain of about 1 or  $2^{14,15}$ . Hence, the knowledge of linear properties and of a few elongational data at low strain rates can provide qualitative indications on the behaviour at higher strain rates.

#### CONSTITUTIVE MODELLING

When considering the flow of polymer melts within the framework of continuum mechanics, extra-stresses T, velocity u and pressure p are calculated. For the sake of

facility, we purposely omit temperature effects. With the assumption of incompressibility, the momentum and mass conservation equations are given by

$$-\nabla p + \nabla \cdot \boldsymbol{T} = \rho \frac{D\boldsymbol{u}}{Dt}$$
(5)

 $\nabla \cdot \boldsymbol{u} = 0 \tag{6}$ 

where  $\rho$  is the fluid density. These partial differential equations require appropriate boundary conditions. In simple words, the boundary conditions describe the process that is considered as well as the flow domain. For example, an extrusion process involves an assigned flow rate at the inlet, vanishing velocities along rigid walls, a stress free jet (extrudate) and a stress free exit. Film casting<sup>26</sup> involves nearly the same conditions, except at the exit where a take-up velocity is applied.

As can be seen, Eqs. 5 and 6 do not suffice for solving the unknowns T, u and p. This is obvious if one considers that they do not invoke any property of the polymer melt under consideration. This is the purpose of the constitutive equation, which, to some extent, describes the melt. In the literature mechanics<sup>2,7,27</sup>, continuum various on families of constitutive equations are found. Differential and integral equations are actually different formalisms for describing a same physical concept: memory effects. Presently, we focus on differential equations, as they are usually easier to handle in a simulation software.

In order to cope with various time scales experimentally observed in relaxation mechanisms, the extra-stress tensor T can be written as a sum of N individual contributions  $T_i$ :

$$\boldsymbol{T} = \sum_{i=1}^{N} \boldsymbol{T}_i \tag{7}$$

In Eq. 7, all  $T_i$  involve directly or indirectly differential relationships. A series of differential viscoelastic models are of the

Oldryod type<sup>28</sup>. They obey a differential equation written in terms of extra-stress  $T_i$  and whose general form is given by:

$$\boldsymbol{T}_{i}\boldsymbol{G}(\boldsymbol{T}_{i}) + \lambda_{i} \frac{\delta \boldsymbol{T}_{i}}{\delta t} = \eta_{i} \left( \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{T} \right)$$
(8)

In Eq. 8,  $\lambda_i$  and  $\eta_i$  are material parameters, namely a relaxation time and a viscosity factor. Here, we easily understand that considering several modes in Eq. 7 endows the rheological model with a relaxation spectrum. The (scalar or tensorial) function  $G(T_i)$  depends on the selected viscoelastic model, and carries non-linear attributes. Finally, the term  $\delta/\delta t$  is a time-derivative operator (contravariant, covariant, etc.), which satisfies the basic objectivity and invariance requirements.



Figure 1. Dynamic and viscometric properties of LDPE 6411<sup>33</sup>. Experimental data (symbols) and model properties including the prediction of normal stress differences (continuous lines).

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It is reasonable to assume that all modes obey the same differential equation, with their own parameters. Next to the Oldroyd model, we find the Johnson-Segalman<sup>29</sup>, Phan Thien-Tanner<sup>30,31</sup> and Giesekus<sup>32</sup> models. It is worth mentioning that the viscometric and rheometric properties of these models depend on a limited number of real parameters. In Fig. 1, we plot the linear and viscometric properties of a LDPE melt together with their model counterparts obtained with a four-mode Giesekus model. As can be seen, a good match is obtained.

Next to models of the Oldroyd type, there exist viscoelastic models for which the extra-stress tensor T is a function of configurational quantities, which in turn obey differential equations. The Leonov model involves a deformation tensor and a potential energy function<sup>34</sup>. In the late nineties, a model for branched polymers, based on reptation tube theory, has emerged, and is referred to as the pom-pom  $model^{35}$ . For each relaxation mode, the extra-stress contribution involves an orientation tensor and a stretching scalar. Both these configurational quantities obey differential equations and involve material parameters. The model has been the object of several studies and improvements<sup>36-38</sup>.

#### NUMERICAL SIMULATION

In most situations, the flow governing equations involve non-linearities, which at first originate from the selected constitutive model. Additional non-linearities originate from the boundary conditions. The most significant one is the so-called free surface, whose mathematical expression involves velocity and geometric attributes.

In view of the non-linearities, the set of governing equations cannot be solved analytically, except for a few cases<sup>39</sup>. Therefore, numerical simulation tools are invoked. Textbooks have already been dedicated to the topic, such as<sup>40,41</sup>. In a few words, the idea consists successively of discretising the flow domain (e.g. with finite elements or finite volumes), building and assembling the system of equations, and solving the non-linear system with an appropriate solver.

Obviously the discretised system is affected by the non-linearities initially existing. In particular, the combination of non-linearities, stress convection and geometric singularities can lead to the well high Weissenberg known number  $problem^{42,43}$ . The most frequently observed symptom is the development of spurious which numerical oscillations in the discretisation grid can often be identified<sup>44</sup>. Within the context of finite elements, the numerical origin of the HWNP has been evidenced<sup>45</sup>. In general, the selection of appropriate discretisations and algorithms allows circumventing possible difficulties<sup>46-50</sup>. But care should also be taken when selecting the rheological model. This last item is certainly of interest: this has allowed the production of simulation results in agreement with experimental data for relatively complex flows<sup>33,51-54</sup>.

# MODEL PROPERTIES AND

### PARAMETER IDENTIFICATION

Based on the above sections, one may face the *a priori* difficult question of selecting the best constitutive equation with the best appropriate material parameters. This question can be split into subquestions, successively related to the number of modes, the constitutive model and the material parameters. Of course, numerical considerations may interfere in this procedure.

A preliminary ingredient concerns the flow kinematics. Indeed, the knowledge of the dominating kinematics contribution allows assigning a priority to the various properties considered. For example, shear effects mainly dominate in extrusion flow: properties, shear viscosity linear and possibly first normal stress differences are considered at first. Elongational effects mainly dominate in film casting and fibre spinning: linear properties and transient elongational viscosity are considered at first. Moderate elongation develops in blow moulding, so that linear properties may often be sufficient. The dominant kinematics character should be quantified: via a typical wall shear rate  $\dot{\gamma}_w$  in extrusion,

or a typical strain rate  $\dot{\epsilon}_f$  in film casting or fibre spinning.

This information already indicates the type of measurements to be performed as well as the corresponding range. This also quantifies the respective importance of the various properties involved. If a singlemode viscoelastic model is selected, an appropriate relaxation time should be of the order of  $1/\dot{\gamma}_{W}$  or  $1/\dot{\epsilon}_{f}$ . Indeed it is reasonable to consider that the melt response time to a solicitation is primarily in accordance with the reciprocal kinematics If a multi-mode viscoelastic intensity. model is selected, it is sensible to consider relaxation times on both sides of  $1/\dot{\gamma}_{w}$  or  $1/\dot{\epsilon}_{f}$ , for a similar reason. Of course, models with five or ten modes can formally be considered<sup>52</sup>; the relevance has to be evaluated with respect to the objectives.

The selection of a particular constitutive model may be based on the amount of available experimental data. When focusing only on shear or extensional properties, the Giesekus model is a good candidate<sup>20,33</sup>. When all properties are needed, the Phan Thien-Tanner or pom-pom models can be considered, since they involve further parameters<sup>52-54</sup>. They can also be selected for the simulation of extensional flows when only qualitative information is available on the extensional viscosity<sup>5</sup>.

Finally, numerical values have to be assigned to the various model parameters. We assume that values for relaxation times have been selected, as suggested above. When a single-mode model is considered, the few remaining parameters should be selected in order to obtain the required properties in the relevant range of kinematics. This can be done e.g. along with a methodology suggested in<sup>55</sup>. When a multi-mode model is selected, viscosity parameters should preferably be selected in order to reproduce the linear properties in the range of interest 56,57. Non-linear parameters are subsequently determined in

order to endow the selected model with the required non-linear properties, in the relevant range of kinematics<sup>20,33,52,53</sup>. However, parameter identification should not be turned into a mathematical fit, while computational constraints should be kept in mind.

#### SELECTED NUMERICAL

#### SIMULATIONS AND VALIDATION

The above considerations have been successfully applied to several viscoelastic flow situations characterised by different kinematics attributes. Hereafter we intend to review and summarise some of these case studies.



Figure 2. Experimental set-up for the analysis of secondary motions in straight and tapered channels: extruders, feedblock and channel<sup>33</sup>. © Society of rheology.

#### Secondary flows in channels

It is established that the second normal stress difference is responsible for the development of secondary motions in viscoelastic flows through non-circular channels. Early experimental investigations were performed e.g. by Giesekus<sup>58</sup>. Various numerical simulations have been carried out, and comparisons with experiments have also In a recent work<sup>33</sup>, been produced. secondary motions are investigated in straight and tapered channels with a square cross-section. The channel length is about 60 cm, while the cross-section height is about 1 cm for the straight channel and decreases from 1 to 0.5 cm for the tapered The secondary motions are channel.

experimentally identified by recording the deformation of the interface between two LDPE melt layers with different pigmentations. In Fig. 2, we display a sketch of the experimental device.

The present flow is mainly dominated by shear effects, and a model is identified on the basis of linear properties, as displayed in Fig. 1. In particular, Eq. 1 is applied for estimating the shear viscosity. A four-mode Giesekus viscoelastic model<sup>32</sup> is selected for performing 3D finite element simulations<sup>50</sup>. A transport equation is subsequently used for tracking the motion and deformations of both fluid layers, in order to predict these secondary motions.



Figure 3. Comparison between experiments (a1,b1) and predictions (a2,b2) for the secondary motions in straight (top) and tapered (bottom) channels<sup>33</sup>. © Society of rheology.

Experiments and simulations are carried out for straight and tapered channels with square cross-sections. In both cases, secondary motions exhibit similar patterns: from the centre towards the walls, along the walls towards the edges, and from the edges towards the centre. This can be seen in Fig. 3, where a comparison between experiments and predictions shows a good agreement. In Fig. 4, we show a prediction of the interface between both fluid layers.

Such experimental and numerical studies reveal that the control of multi-layer fluid systems can be accompanied by serious difficulties. One of the most typical situations of industrial relevance is the coextrusion flow in a coat hanger die.



Figure 4. Shape of the interface between both fluid layers, for the straight (left) and tapered (right) channels.

#### Transient elongational recovery

The knowledge of melt behaviour in recovery experiments can be relevant for several industrial polymer processing. Although time scales are usually short in actual polymer processing, the acquisition of relaxation data at both short and long time scales can be of interest. Transient elongational recovery is a convenient rheometrical procedure for acquiring such data. The experiment consists of stretching a melt sample at an assigned strain rate. As a specified Hencky strain is reached, the sample is released at one extremity and the transient recovery is measured. In such an experiment, it is remarkable to note that the recovery can develop over a time scale much longer than that of the prior stretching.

Predictions of transient elongational recovery are compared to experimental data<sup>52</sup> for a well characterised HDPE melt. An extensive rheological characterisation of the melt is carried out: linear moduli, shear viscosity and transient elongational viscosity are measured. Based on this, a multi-mode Phan Thien-Tanner viscoelastic

model<sup>30,31</sup> is selected. Although nine modes are considered, five modes may probably be sufficient. Viscosity factors are identified from linear properties, while non-linear parameters are successively determined from shear and elongational viscosities. In Fig. 5, we plot the rheological properties of the HDPE melt and the model counterparts. As can be seen, a good match is found.



Figure 5. Linear properties (top) and transient elongational viscosity (bottom) for a HDPE melt<sup>52</sup>. Experimental data (symbols) and model properties (lines). © Springer Verlag.

The simulations of transient elongational recovery are performed under the same conditions the as corresponding experiments. Starting from rest state, the sample is stretched, released and recovery is In Fig. 6 we show a recorded vs. time. comparison between measurements and simulations for recovery experiments performed under various stretching conditions. The recovery is defined as the ratio of the initial sample length  $L_0$  to the current one L(t). As can be seen, a good agreement is found between calculations and data. As stated above, it is interesting to note that the recovery develops at least up to  $10^3$  s, although stretching time is of the order of the second. At distant time scales, when viscoelastic stresses are nearly fully relaxed, surface tension starts to play a role.



An important remark must be added. This application has been simulated with accuracy requirements that are probably beyond industrial relevance. Of course, relaxation mechanisms and shape recovery do occur in extrusion process; however, they are quickly frozen on a production line. In a way, the present experiments and simulations can certainly serve rheological objectives. for macromolecular e.g. characterisation.

#### Stresses in contraction flow

Most industrial viscoelastic flows occur in geometries that exhibit abrupt crosssection changes. It is therefore interesting to consider the well-defined 4/1 contraction flow, for which an abundant literature on experimental and modelling aspects exists. Next to the analysis of flow patterns, such a flow case also allows evaluating the performances of constitutive models as well as of simulation softwares.



Figure 7. Measurements (symbols) and model predictions of transient elongational viscosity of a LDPE at various strain rates<sup>53</sup>. © Elsevier.

In planar flow situations, when the melt is transparent, birefringence techniques allow evaluating the principal stress difference, by applying the stress-optical rule. Again, a comparison with predictions is interesting. This is performed for the contraction flow of melt a LDPE characterised by a moderate branching level<sup>38,53</sup>. Here too, extensive rheological measurements are carried out, and the melt is described with a four-mode viscoelastic pom-pom model<sup>37,53</sup>. In particular, a series of non-linear parameters, characterising the branching level, are determined from the elongational viscosity. Fig. 7 displays the measurements of the transient elongational viscosity at various strain rates, as well as the model counterparts. A deviation is found at low strain rate, it results from the absence of a very long relaxation time in the model.

Birefringence photographs provide only a series of dark and light streaks. A conversion is needed for evaluating the difference principal stress from birefringence and vice-versa. An appropriate calibration on the basis of a well-defined flow field (Poiseuille flow) enables identifying the stress optical coefficient<sup>53</sup>. A comparison between predictions and birefringence measurements at a relatively high flow rate is shown in Fig. 8. A good

agreement is found for the development of fringes in the vicinity of the contraction, despite possible uncertainties near the reentrant corner.



Figure 8. Experimental (top) and predicted fringes for the contraction flow at a high downstream wall shear rate<sup>53</sup>. © Elsevier.

Fig. 9 shows the stress development along the symmetry line of the contraction and along a line close to the downstream wall. Experimental data on the principal stress difference are obtained via a careful counting of fringes. Here also, a good agreement is found. It must be noted however that no reliable experimental data can be obtained close to the wall, where fringes are mainly oriented along the flow direction. In general, the relevance of such a calculation is that it enables the prediction of stress levels as well as of secondary motions (possible dead zones).



Figure 9. Experimental and calculated principal stress difference along the central symmetry line and along a line close to the downstream wall of the contraction<sup>53</sup>. © Elsevier.

#### Coextrusion film casting

Numerical simulation can also be applied to the analysis and optimisation of actual industrial flows. For this purpose, we select the coextrusion film casting<sup>5</sup>, where narrow LDPE melt stripes are used for the production of thin LLDPE films. The concept is illustrated in Fig. 10, where we see a portion of the film with the white LDPE stripe, between the slit die exit and the contact on the chill roll.



Figure 10. Coextrusion film casting: close view of the film between the die exit and the contact with the chill roll indicated with the arrow.

From the point of view of geometry, the film is characterised by a dimension that is several orders of magnitude lower than the other ones. For this purpose, a thin film model is used, where the thickness becomes an unknown together with the velocity and stresses<sup>26</sup>. It is worth mentioning that the flow kinematics in film casting is essentially dominated by elongation.

From the point of view of rheology, both melts are characterised in the linear regime. In particular, no data is available on the extensional viscosity. The inspection of linear properties shown in Fig. 11 reveals that the LDPE exhibits a more pronounced shear thinning behaviour than the LLDPE. In addition, from general knowledge, it is accepted that LDPE melts exhibit a significant strain hardening behaviour in uniaxial extension, which originates from branching. These data and considerations allow determining a four-mode Phan Thien-Tanner fluid model<sup>30,31</sup> for both melts. Viscosity factors are determined from the linear properties, while the parameters controlling the elongational viscosity are selected in accordance with the qualitative information in extension.



LLDPE melts used in coextrusion film casting.

Experiments are performed at various melt flow rates and take-up velocities while transverse thickness profiles are measured. Numerical simulations are performed under the same conditions, and thickness profiles are also recorded. In Fig. 12, we report the measured and predicted thickness profiles vs. the distance from the film edge. Again, a good agreement is found. This result is remarkable, when considering the assumptions made and the basic knowledge on the rheological melt properties.



Figure 12. Profiles of measured and predicted transverse thickness h vs. the distance x from the film edge.

#### CONCLUSIONS

Simulation of polymer processing remains an activity endowed with various aspects: property measurement, modelling and assumptions. Interestingly, despite the geometric simplicity of flow domains involved, relatively complex melt behaviours are found. They mainly originate from intrinsic material attributes, and a property develops according to the prescribed boundary conditions.

A few applications have been shown, with a comparison against experimental data. A good quantitative agreement has often been found. This is certainly true and easy to admit when a very refined rheological model is used. But relevant predictions are also found when the selected model is endowed with uncertainties, originating from a lack of relevant data.

It is probably important to realise that the availability of constitutive equations endowed with required properties, such as shear thinning, strain hardening, is of great help. Beyond that, a careful identification of kinematics features, good engineering feeling, awareness of potentialities and limitations of data, models and solvers are necessary ingredients for successful predictions.

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