Direct Conversion of Creep Data to Dynamic Moduli

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The conversion of creep compliance to dynamic moduli is one of the promising ways to overcome the limitation of the dynamic measurements for wide range of frequency. Although Evans et al.[1] developed a direct conversion method using numerical differentiation, which is very weak for the errors in data (Figure 1). It is known that numerical differentiation is a representative ill-posed problem[2]. Kim et al.[3] suggested a more stable method one using numerical integration (the Laplace transform). They used the Havriliak-Negami (HN) model[4] to fit the Laplace transform of creep compliance, P(s).

$$\Pi(s) = \frac{1}{\eta_o s} + \frac{J_m}{\left[1 + (\tau_m s)^{\alpha}\right]^{\beta}} \tag{1}$$

where h_o, J_m, t_m, a and b are the parameters to be determined by regression of P(s). However, the method of Kim et al.[3] can be applicable only to specific materials which fit to the HN model. Besides, we found that the HN model cannot recover the parameters through analyzing the simulated data (Table 1). The aim of this study is to improve the method of Kim et al.[3] by introducing two new approximate functions instead of the HN model. One is the logarithmic Chebyshev polynomial as follows:

$$\Pi(s) = \exp\left[\sum_{n=0}^{N} j_n T_n(\xi)\right]$$
$$\text{where} \quad \xi = \frac{\log s - \log\sqrt{s_{max} s_{min}}}{\log\sqrt{s_{max}/s_{min}}} \tag{2}$$

j_n, T_n, N, s_{min} and s_{max} represent the Chebyshev coefficient of the n^{th} order, the Chebyshev polynomial of the n^{th} order, the order of polynomial, the minimum and maximum values of s, respectively. The other is the piecewise linear (PWL) function such as

$$\Pi(s) = \frac{1}{\eta_o s}\prod_{k=1}^{K}\left[1 + (\tau_k s)^{\alpha_k}\right]^{\beta_k} \tag{3}$$

where K denotes the number of piecewise lines. The PWL functions are similar to the spline functions and the parameters, t_k, a_k and b_k, relate to the change of the slope in the plot of log P(s) against log s. For both functions, substitution of iω to s enables us to decompose P(s) into the real and imaginary parts as dynamic compliances. We tested the methods to the simulated data (viscoelastic fluid and solid) generated by the HN model (Figure 1) as well as the experimental data (Figure 2) and compared the results with those of previous methods. Consequently, we concluded that the polynomial method is better than the others according to following criteria: (1) the approximate functions should be identified consistently, (2) stability for experimental error is demanded, (3) the algorithm should be flexible for any type of creep compliance, and (4) the algorithm should give the consistency with measured dynamic moduli.

Table 1. Comparison of the HN parameters obtained from regression to those used for data generation.

Parameter	Viscoelastic fluid		Viscoelastic solid	
	Original	Regression	Original	Regression
η_o	37	36		
J_m	0.15	0.075	0.15	0.14
τ_m	28	2.4	28	19
α	0.42	0.55	0.42	0.47
β	1	1	1	1

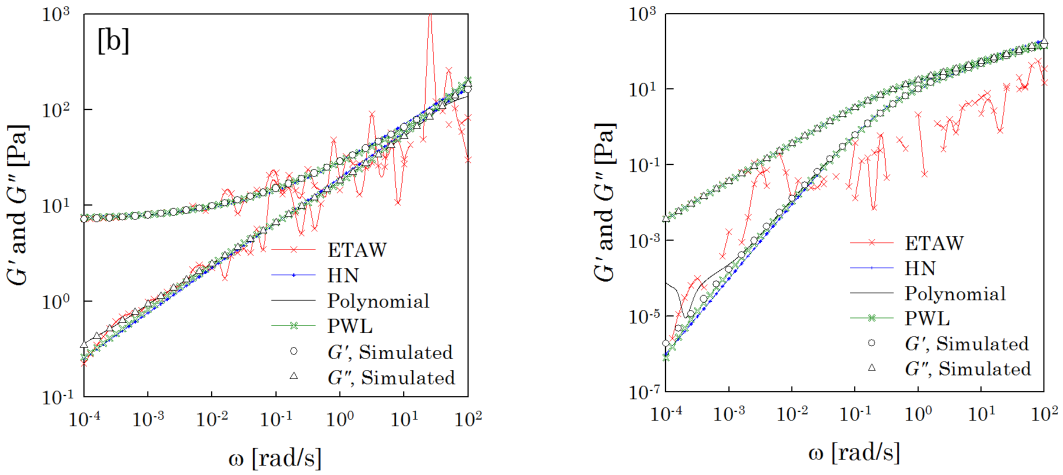

Figure 1. Dynamic moduli of [a] viscoelastic fluid and [b] solid. Lines are the conversion results of four methods using 1% error-contaminated creep compliances: the ETAW, the HN, the polynomial and the PWL methods. Symbols represent simulated moduli.

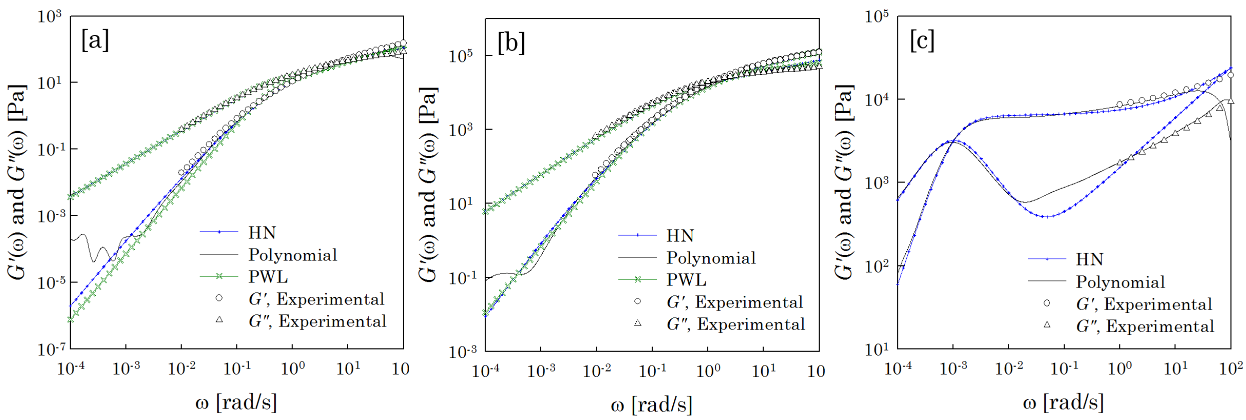

Figure 2. Dynamic moduli of three materials: [a] aqueous solution of polyethylene oxide (PEOs), [b] Polystyrene melt (PSm) and [c] cellulose/water/[EMIM]Ac system. Symbols represent the dynamic moduli measured experimentally and lines are the conversion results: the HN, the polynomial and the PWL method.

ACKNOWLEDGEMENT
This work was supported by the Mid-Career Researcher Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2013R1A1A2055232). It was also supported by the DGIST R&D Program of the Ministry of Science, ICT and Technology of Korea (15-NB-02).

REFERENCES

1. Evans, R. M. L., Tassieri, M., Auhl, D., and Waigh, T.A. (2009), "Direct conversion of rheological compliance measurements into storage and loss moduli", *Phys. Rev. E,* **80**, 012501.

2. Cullum J. (1971), "Numerical Differentiation and Regularization", *SIAM. J. Numer. Anal.,* **8**, 254-265.

3. Kim, M., Bae, J-E., Kang, N., and Cho, K.S. (2015), "Extraction of viscoelastic functions from creep data with ringing", *J. Rheol.,* **59**, 237-252.

4. Havriliak, S. and Negami.S. (1967), "A Complex Plane Representation of Dielectric and Mechanical Relaxation Processes in Some Polymers", *Polymer,* **8**, 161-210.