

## Viscosity estimation of Newtonian liquids from data obtained by oscillation-type density meters

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### ABSTRACT

This communication evidences the possibility to use data obtained by oscillation-type density meters, Anton Paar DMA 5000 M, to estimate the viscosity value of Newtonian liquids.

In this type of density meters, the viscous behaviour of the liquid during the oscillation leads to a damping and by this to a lower resonance frequency of the measuring system and, thus, to an incorrect density indication. However, these instruments have been developed to overcome this error by applying a viscosity-related correction. Among other outputs, the density indication values without viscosity correction  $d_{nc}$  and with viscosity correction  $d$  and a damping indication parameter  $Q$  are given by these density meters in each measurement performed.

It has been proved to be possible to estimate the viscosity of the Newtonian liquids tested, at 20 °C and 23 °C, in the viscosity interval from 7 mPa s to 220 mPa s, with a relative standard uncertainty of 3 %, by using a third-order polynomial regression of the kinematic viscosity against the difference of the density indication values with and without viscosity correction given by a DMA 5000 M. A second more simple possibility is to calculate the viscosity from the damping indication parameter  $Q$ .

These findings might be considered being useful for laboratories that need a rough estimation of sample viscosity without having the opportunity to measure the viscosity directly by other methods.

### INTRODUCTION

Liquids viscosity can be measured by means of various methods<sup>1</sup>, like capillary, falling ball, Stabinger or rotational viscometers, and others. Despite the wide availability of viscosity measurement methods, in terms of accuracy, method complexity and costs, not all laboratories possess one of these instruments, especially when they only need a rough estimation of sample viscosity.

Likewise, a large variety of measuring instruments and methods can also be used to measure the density of the liquids<sup>2</sup>. In the last decades, the oscillation-type density meters have shown great versatility and they have been used in various branches of industry. The advantages of using these measuring instruments are mainly related with their simplicity in use, a wide density and temperature measuring interval, a fast measurement time, and small sample volumes needed.

The working principle of an oscillation-type density meter is based on the law of harmonic oscillation. In this type of measuring instrument, a U-shaped tube is filled with the sample. This measuring cell

containing the sample will act like a flexural oscillator; the density of the liquid is determined by measuring the period of oscillation. The viscous behaviour of the liquid during the oscillation leads to a damping and by this to a lower resonance frequency of the measuring system and, thus, to an incorrect density indication. Damping detection is performed by analysing the frequency of harmonics of the oscillation. From these data the density meter is determining the not viscosity-corrected density value  $d_{nc}$  and a viscosity-corrected density value  $d$ . These data are given together with a damping indication parameter  $Q$ , a dimensionless quantity<sup>3</sup>.

In this work, the possibility of estimating the viscosity of a Newtonian liquid from data measured oscillation-type density meters, DMA 5000 M from Anton Paar, was evaluated.

The method to calculate the density correction is not published in detail, thus, a physical model cannot be used for the calculation of viscosity. Instead of this mathematical approximations have been used.

## EXPERIMENTAL PROCEDURE

### Materials

Two oscillation-type density meters DMA 5000 M from Anton Paar (referred as (1) and (2)) were tested at a measuring temperature of 20 °C, the second additionally at 23 °C, with 13 Newtonian liquids (NL): ultra-pure water<sup>4</sup>, n-nonane, and 11 oils (mineral and poly-alpha-olefin (PAO) oils). The dynamic viscosity  $\eta$  of the tested liquids, at 20 °C, ranged from 0.70 mPa·s to 220 mPa·s.

The dynamic viscosity value of ultra-pure water was taken from ISO TR 3666<sup>5</sup>. For the other test liquids, the kinematic viscosity values were measured with capillary viscometers<sup>6,7</sup>. The dynamic viscosity was calculated using these values and the density values, measured by means

of a hydrostatic weighing method<sup>8</sup>. The expanded uncertainty of these measurements is 0.20 % (coverage factor  $k = 2$ , 95 % level of confidence).

The uncertainty of the dynamic viscosity values was obtained according to GUM methodology<sup>9</sup>. This uncertainty is negligible compared to the uncertainty of the viscosity estimated by density measurements as shown in the following.

### Viscosity estimation from damping indication parameter

The sample viscosity effect on the DMA 5000 M damping indication parameter  $Q$ , was studied at 20 °C and 23 °C for samples with dynamic viscosity  $\eta$  in the interval from 0.70 mPa·s to 220 mPa·s, using the liquids described above.

### Viscosity estimation from density indication

Both density indications of the density meter, the density not viscosity-corrected  $d_{nc}$  and the density viscosity-corrected  $d$ , were analysed. The difference between these two density indications  $d_{nc} - d$ , for simplification denoted from now on as  $D$ , was studied using the same premises of the previous point.

## RESULTS FOR SINGLE DEVICES

### Measured data

The data measured at 20 °C and 23 °C with two different density meters (1) and (2) is given in the Table 1. The  $Q$  values are taken directly from the DMA indication and the  $D$  values are calculated as described above (Table 1).

### Viscosity estimation from damping indication parameter

During the study of the relation between viscosity  $\eta$  and the damping indication parameter  $Q$ , it was observed that the lower the viscosity of the sample the higher was the damping indication parameter given by the density meter. To the highest damping

indication (~2845) corresponded the lowest viscosity value tested (~0.7 mPa·s) and to the lowest damping parameter value (~1023) corresponded the highest viscosity value tested (~220 mPa·s) (Table 1). This seems to be an inverse relation between the expected physical damping of the oscillation and the damping indication parameter value  $Q$  given by the density meter.

Table 1. Resume of results of  $Q$  and  $D$ , in  $\text{kg/m}^3$ , obtained at 20 °C and 23 C by density meters (1) and (2).

$\eta$ mPa·s	Density meter (1)		Density meter (2)	
	$Q_{(1)}$	$D_{(1)}$ $\text{kg/m}^3$	$Q_{(2)}$	$D_{(2)}$ $\text{kg/m}^3$
	20 °C			
0.71	2844.9	-0.003	2760.2	-0.006
1.00	2761.2	0.000	2697.1	-0.004
1.45	2667.8	0.016	-	-
2.87	2468.7	0.042	-	-
7.68	2137.9	0.097	2075.5	0.095
10.03	2032.5	0.119	-	-
19.75	1774.3	0.184	-	-
31.70	1601.8	0.233	-	-
60.06	1386.1	0.317	1356	0.323
101.48	1224.9	0.399	-	-
125.00	1171.3	0.429	1133.9	0.440
224.06	1023.1	0.546	-	-
23 °C				
0.68	-	-	2745.18	-0.006
0.93	-	-	2685.97	-0.005
6.94	-	-	2094.03	0.089
27.72	-	-	1597.13	0.217
51.75	-	-	1400.33	0.300
106.87	-	-	1169.18	0.414

It was found that the dynamic viscosity  $\eta$  of the Newtonian samples versus damping indication parameter  $Q$ , in the interval described above, at 20 °C and 23 °C, can be described by a simple exponential regression for both density meters tested ((1) and (2)) with an accuracy which is satisfying for many approaches. This is shown in Fig. 1. The coefficients of the approximation and the relative standard

deviations  $s$  of the regressions are given in Table 2.

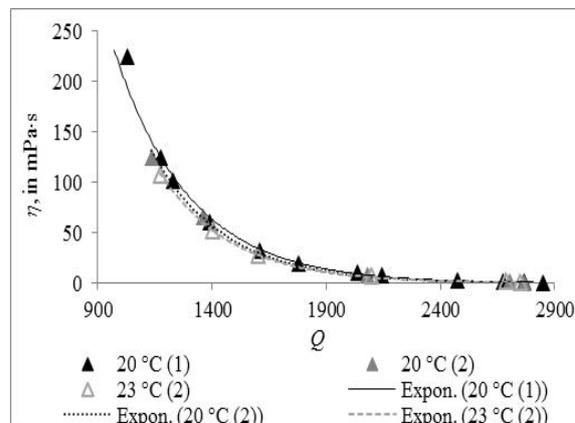


Figure 1. Regression curve of the dynamic viscosity  $\eta$ , in  $\text{mPa}\cdot\text{s}$ , at 20 °C and 23 °C, against  $Q$  measured by density meters (1) and (2).

Dynamic viscosity estimation from density indications

The results of the difference between the density indication values not viscosity-corrected and viscosity-corrected,  $D$ , can be well described by polynomial regressions of third order for both density meters tested ((1) and (2)). The data are visualized in Fig. 2.

The coefficients of the approximation and the relative standard deviations  $s$  of the regressions are given in Table 3.

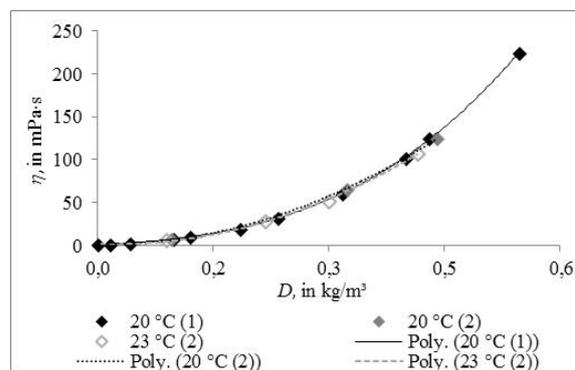


Figure 2. Regression curve of the dynamic viscosity  $\eta$ , in  $\text{mPa}\cdot\text{s}$ , at 20 °C and 23 °C, against  $D$ , in  $\text{kg/m}^3$ , measured by density meters (1) and (2).

Table 2. Resume of the coefficients (*a* and *b*) and relative standard deviations *s* of the regressions of  $\eta$ , in mPa·s, against *Q*, at 20 °C and 23 °C, in the viscosity interval from 0.7 mPa s to 220 mPa·s, obtained by density meters (1) and (2).

<i>t</i> °C	Density meter	Regression coefficients ( <i>a</i> and <i>b</i> ) of $\eta(Q) = a e^{-b Q}$		<i>s</i> % ( <i>k</i> =1)
		<i>a</i>	<i>b</i>	
20	(1)	4339	-0.00302	9
	(2)	4670	-0.00314	8
23	(2)	4425	-0.00316	8

Table 3. Resume of the coefficients (*a*, *b*, *c*, and *d*) and relative standard deviations *s* of the regressions of  $\eta$ , in mPa·s, against *D*, in kg/m<sup>3</sup>, at 20 °C and 23 °C, in two viscosity intervals (V1: 0.7 mPa s to 7 mPa·s, V2: 7 mPa s to 220 mPa·s), obtained by density meters (1) and (2).

<i>t</i> °C	Density meter	Regression coefficients ( <i>a</i> , <i>b</i> , <i>c</i> , <i>d</i> ) of $\eta(D) = aD^3 + bD^2 + cD + d$				<i>s</i> % ( <i>k</i> =1)	
		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	V1	V2
20	(1)	917	174	41	1	9	1.7
	(2)	390	404	29	1	15	0.3
23	(2)	822	163	47	1	11	1.3

Regressions residual analysis

In Fig. 4 and Fig. 5 the residuals of these approximations are shown.

For the *Q* regression the maximum relative deviation in viscosity observed was 15 % (Table 4). The residuals data follow a curvature clearly to be seen (Fig. 4). This implies that another approximation curve could yield better results. Here we abstained from a more complex approximation to have the opportunity of getting a simple approximation.

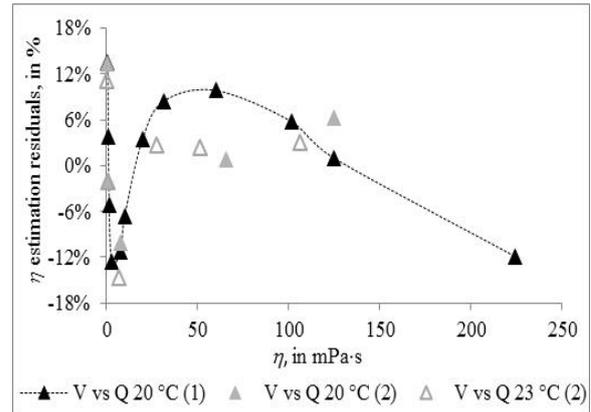


Figure 4. Residuals analysis of the regressions curves of dynamic viscosity  $\eta$  (V) against *Q*, measured by density meters (1) and (2), at 20 °C and 23 °C.

Table 4. Resume of the relative residuals, in %, of the regression equations of dynamic viscosity  $\eta$  against *Q* and against *D*, obtained at 20 °C and 23 °C, by density meters (1) and (2).

$\eta$ mPa·s	Density meter (1)		Density meter (2)	
	<i>Q</i> <sub>(1)</sub> %	<i>D</i> <sub>(1)</sub> %	<i>Q</i> <sub>(2)</sub> %	<i>D</i> <sub>(2)</sub> %
	20 °C			
0.71	14	24	13	18
1.00	4	0	-2	-11
1.45	-5	18	-	-
2.87	-13	8	-	-
7.68	-11	-3	-10	1
10.03	-7	-1	-	-
19.75	3	2	-	-
31.70	8	0	-	-
60.06	10	1	1	0
101.48	6	2	-	-
125.00	1	-2	6	0
224.06	-12	0	-	-
	23 °C			
0.68	-	-	-2	-17
0.93	-	-	11	5
6.94	-	-	-15	2
27.72	-	-	3	-1
51.75	-	-	2	0
106.87	-	-	3	0

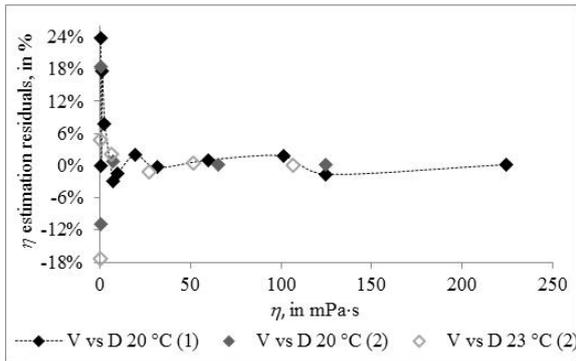


Figure 5. Residuals analysis of the regressions curves of dynamic viscosity  $\eta$  (V) against  $D$ , measured by density meters (1) and (2), at 20 °C and 23 °C.

Looking for the  $D$  approximation using a third-order polynomial one can find comparably larger deviations only for viscosities below 7 mPa·s, the maximum relative deviation was here 24 % (Table 4). In the viscosity interval of 7 mPa·s to 220 mPa·s, the viscosity can be determined from  $D$  values with a maximum deviation lower than 3 % (Table 4).

The residuals obtained from the simple exponential regression of  $(\eta, Q)$  pairs of values, in the same viscosity interval, implies, in average, errors 9 times higher than the ones obtained with the third-degree polynomial regression of  $(\eta, D)$  pairs of values (Table 4).

#### COMBINED RESULT FOR TWO DEVICES

To use a calculation procedure for the determination of the viscosity as described above, the results variation due to instrumental differences has to be known. Thus, the study was performed at two different Anton Paar DMA 5000 M, mentioned as (1) and (2).

The data gained using both devices were put together and one approximation was calculated using all data. The results of this approximation are given in Tables 5 and 6, being equivalent to the Tables 2 and 3,

which are for single devices. In Table 7 the residuals to this combined approximation are given.

Table 5. Resume of the coefficients ( $a$  and  $b$ ) and relative standard deviations  $s$  of the regressions of  $\eta$ , in mPa·s, against  $Q$ , at 20 °C and 23 °C, in the viscosity interval from 0.7 mPa s to 220 mPa·s, for the combined data set obtained by density meters (1) and (2).

$t$ °C	Regression coefficients ( $a$ and $b$ ) of $\eta(Q) = a e^{-b Q}$		$s$ % ( $k=1$ )
	$a$	$b$	
20	4413	-0.00308	13
20, 23	4413	-0.00308	15

Table 6. Resume of the coefficients ( $a, b, c$  and  $d$ ) and relative standard deviations  $s$  of the regressions of  $\eta$ , in mPa·s, against  $D$ , in kg/m<sup>3</sup>, at 20 °C and 23 °C, in two viscosity intervals (V1: 0.7 mPa s to 7 mPa·s, V2: 7 mPa s to 220 mPa·s), for the combined data set obtained by density meters (1) and (2).

$t$ °C	Regression coefficients ( $a, b, c, d$ ) of $\eta(D) = aD^3 + bD^2 + cD + d$				$s$ % ( $k=1$ )	
	$a$	$b$	$c$	$d$	V1	V2
20	1037	86	53	1	20	3.1
20, 23	1089	45	59	1	21	3.1

Using the data obtained for both density meters, the regression curves to obtain the dynamic viscosity  $\eta$  from  $Q$  and  $D$  values where plotted in Fig. 6 and 7, respectively, together with the deviations from the directly measured viscosity (right axis).

In the exponential regression for the data from both density meters tested a relative standard deviation in viscosity of 13 % was observed for the entire viscosity interval tested, i.e. from 0.7 mPa·s to 220 mPa·s (Table 5). This value increased to 15 % when including the 23 °C data.

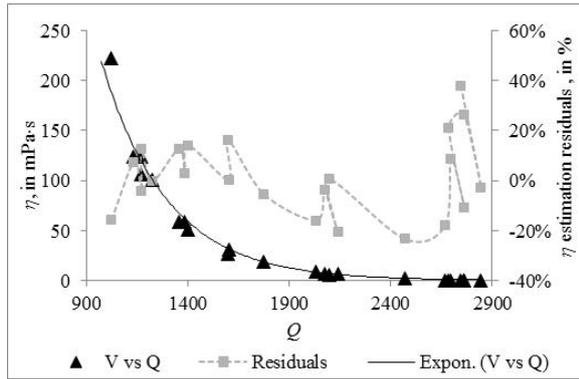


Figure 6. Curve of the regression of dynamic viscosity  $\eta$  (left axis) against the  $Q$  obtained for the combined data set of density meters (1) and (2), at 20 °C and 23 °C, and residuals curve of dynamic viscosity  $\eta$  estimation, in %, from the exponential regression curve (right axis).

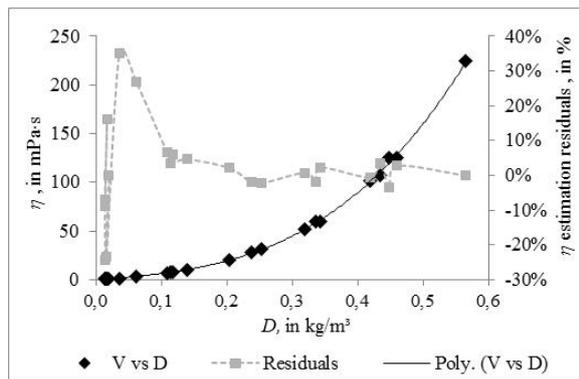


Figure 7. Curve of the regression of dynamic viscosity  $\eta$  (left axis) against the  $D$  obtained for the combined data set of density meters (1) and (2), at 20 °C and 23 °C, and residuals curve of dynamic viscosity  $\eta$  estimation, in %, from third-degree polynomial curve (right axis).

It was seen that a third-degree polynomial regression of viscosity against  $D$  can estimate the viscosity values in  $D$  interval from 0.097 kg/m<sup>3</sup> to 0.546 kg/m<sup>3</sup>, corresponding to a viscosity interval from 7.68 mPa·s to 224 mPa·s, with a relative standard deviation of 3.1 %. For viscosity values from 0.7 mPa·s to 7 mPa·s, corresponding to a  $D$  interval from

-0.004 kg/m<sup>3</sup> to 0.042 kg/m<sup>3</sup>, a relative standard deviation in viscosity of 20 % was observed (Table 6). Both standard deviations are nearly unchanged if including the 23 °C data (21 %).

Table 7. Resume of the relative residuals, in %, of the combined regression of dynamic viscosity  $\eta$  against  $Q$  and against  $D$  obtained, at 20 °C and 23 °C, by density meters (1) and (2).

$\eta$ mPa·s	Density meter (1)		Density meter (2)	
	$Q_{(1)}$ %	$D_{(1)}$ %	$Q_{(2)}$ %	$D_{(2)}$ %
20 °C				
0.71	-3	16	26	-9
1.00	-11	0	9	-24
1.45	-18	35	-	-
2.87	-23	27	-	-
7.68	-21	6	-4	3
10.03	-16	5	-	-
19.75	-5	2	-	-
31.70	0	-2	-	-
60.06	3	-2	13	2
101.48	0	-1	-	-
125.00	-4	-4	7	3
224.06	-16	0	-	-
23 °C				
0.68	-	-	38	-7
0.93	-	-	21	-24
6.94	-	-	1	7
27.72	-	-	16	-2
51.75	-	-	14	1
106.87	-	-	13	3

## CONCLUSIONS

In this communication two different approaches were described and tested in order to determine the sample dynamic viscosity value  $\eta$  for Newtonian liquids from data measured by oscillation-type density meters DMA 5000 M from Anton Paar. The study was performed using water, hydrocarbons, mineral and PAO oils, at measuring temperatures of 20 °C and 23 °C, in the viscosity interval from 0.7 mPa·s to 220 mPa·s.

Two different approaches have been tested for the calculation of liquids viscosity. The first approach was to calculate the viscosity from the damping indication parameter  $Q$  and to describe these data by a simple exponential equation. The second approach was based in the calculation of the viscosity from the difference  $D$  in the density indications of the density meter, the not viscosity-corrected density  $d_{nc}$  and the viscosity-corrected density  $d$  and to use a polynomial approximation for the description of the viscosity-density difference relation.

The investigation was performed using two DMA 5000 M to look for the effects of instrumental differences in the measurements results. The results were calculated for each individual device and for both together.

Using the viscosity estimation from  $Q$  it was possible to calculate the viscosity with a relative standard uncertainty of less than 15 % in the viscosity interval from 0.7 mPa·s to 220 mPa·s. This method is usable as a fast estimation, where a precise knowledge of viscosity is not necessary.

The  $D$  approach was able to yield viscosity values with a relative standard uncertainty of 3 % in the viscosity interval from 7 mPa·s to 220 mPa·s, corresponding to  $D$  values in the interval from 0.097 kg/m<sup>3</sup> to 0.546 kg/m<sup>3</sup>.

For lower viscosities in the range from 0.7 mPa·s to 7 mPa·s the same approximation can be used only with a larger uncertainty of 20 %.

The relations are valid for both investigated devices, so it can be assumed that they are valid for any oscillation-type density meter DMA 5000 M.

In all cases the inclusion of the 23 °C data does not increase the standard deviation significantly. Thus, these approximations can at least also be used in measurements performed at 23 °C.

The relations demonstrated in this communication were derived for unbranched hydrocarbons with single and double bonds but containing no other functional groups. Different classes of Newtonian liquids, within the same density and viscosity range, but with different molecular structures, for instance siloxanes or ethers, may give different results. This need to be checked in future.

#### ACKNOWLEDGMENTS

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