Effect of channel confinement on the steady flow of Bingham plastic fluids across a confined square cylinder

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ABSTRACT

The momentum transfer characteristics for the steady flow of Bingham plastic fluids across a square cylinder placed in a channel have been numerically studied in terms of the streamlines, yield surfaces and drag coefficients. The present numerical results on the drag are consolidated via a simple predictive expression.

INTRODUCTION

Bingham plastic fluids constitute a major class of viscoplastic fluids (Macosko¹). These materials are characterized by their dual nature, i.e., coexistence of yielded (fluid-like) and unvielded (solid-like) regions depending upon the existing stress levels vis-a-vis the fluid yield stress thereby making their homogenization and heating/cooling far more complicated than that for simple Newtonian fluids like air and water.

Recently much research work focussed on investigating the momentum/heat transfer characteristics for a circular, square and elliptical cylinder immersed in viscoplastic fluids (Mossaz et al.²; Nirmalkar and Chhabra³; Nirmalkar et al.⁴; Patel and Chhabra⁵). However most of these studies deal with the unconfined flow conditions. The available literature dealing with the effect of confinement on the flow past a cylinder pertains mostly to power-law fluids (Bharti et al.⁶; Gupta et al.⁷; Dhiman et al.⁸) except for the study of Mitsoulis⁹. Mitsoulis⁹ has studied creeping viscoplastic flow past a confined circular cylinder and reported the effect of the Bingham number on drag and on the shape and extent of yielded/unyielded regions. Hence there are no such numerical results available for the flow of viscoplastic fluids past a confined square cylinder. Thus, the aim of the present study is to explore the effect of channel confinement on the steady flow of Bingham plastic fluids past a confined square cylinder for a wide range of conditions as: Reynolds number, $0.1 \le Re \le 40$; Bingham number, $0.01 \le Bn \le 100$; blockage ratio, $\beta = 0.2$, 0.3 and 0.4. Over these range of conditions, the flow field is steady.

PROBLEM FORMULATION

Consider the flow of an incompressible Bingham plastic fluid with inlet velocity, U_0 past a square bar paced in a channel, as shown in Fig. 1. The extent of confinement is defined by the blockage ratio, $(\beta = l/H)$ which varied from 0.2 to 0.4 in this study.



Figure 1. Schematic representation of the flow and computational domain.

The Bingham plastic fluid enters the channel at the uniform inlet velocity, U_0 and eventually develops into the fully developed condition well before impinging on the square cylinder. The flow is assumed to be steady, laminar, 2-D and incompressible. The equations of continuity and momentum in their dimensionless forms are written as follows:

Continuity equation:

$$\nabla . V = 0 \tag{1}$$

Momentum equation:

$$(V \cdot \nabla) V = -\nabla P + \frac{1}{Re} \nabla \cdot \tau \tag{2}$$

In the aforementioned equations, the length, velocity, pressure and the stress components are scaled using l, U_0 , ρU_0^2 and $\mu_b(U_0/l)$ respectively. Evidently the flow field is governed by three dimensionless numbers namely, Bingham number, $Bn = [(\tau_0 l)/(\mu_b U_0)]$, Reynolds number, $Re = [(\rho U_0 l)/(\mu_b)]$ and blockage ratio, β . For a Bingham plastic fluid the extra stress tensor, τ is given as (Macosko¹):

$$\tau = \left(1 + \frac{Bn}{|\dot{\gamma}|}\right)\dot{\gamma}$$
, if $|\tau| > Bn$ (3a)

$$\dot{\gamma} = 0$$
, if $|\tau| \le Bn$ (3b)

where $\dot{\gamma}$ is the rate of strain tensor and $|\dot{\gamma}| = \sqrt{tr(\dot{\gamma}^2)}$. Here τ is related to the scalar viscosity function (η) as:

$$\tau = \eta \dot{\gamma} \tag{4}$$

Evidently, the incorporation of Eq. 3 directly into a numerical scheme is not straightforward due to its discontinuous and non-differentiable nature. One of the most commonly applied regularization approach to overcome this issue is the Papanastasiou's exponential model¹⁰. Using this regularization approach, the scalar viscosity function (η) is re-written as:

$$\eta_{\rm p} = 1 + \frac{Bn \left[1 - \exp\left(-m \left| \dot{\gamma} \right| \right) \right]}{\left| \dot{\gamma} \right|} \tag{5}$$

Here *m* is a dimensionless regularization parameter; obviously, large values of $m \rightarrow \infty$ (in Eq. 5) are required to predict the true Bingham plastic model given by Eq. 3. The problem closure is obtained by specifying the boundary conditions. These are of no-slip on all solid boundaries; the uniform velocity at the inlet and a zero diffusion flux condition for all dependent variables (except pressure) at the exit of the duct are used, as shown in Fig.1.

NUMERICAL SOLUTION SCHEME

The governing differential equations subject to the appropriate boundary conditions have been solved using the Finite-element based solver COMSOL Multiphysics (Version 4.3a). The effective viscosity for the regularized Papanastasiou -Bingham plastic model¹⁰ was estimated via a user defined function (UDF). A relative convergence criterion of 10^{-5} is used for the primitive variables (p, v). Further the von Mises criterion (Macosko¹) with a relative tolerance level of 10⁻⁶ is used for approximating the yield surfaces. Furthermore a fine mesh is created using quadrilateral cells with non-uniform spacing in the regions of high gradients (close to the cylinder surface and near the confining walls) as well as near the yield surfaces so as to capture the steep velocity gradients even at the highest value of the Reynolds number, i.e., Re = 40 used here. Next, in order to ensure that the present results are free from domain effects, the effect of the upstream (L_u) and downstream lengths, (L_d) on the total drag and pressure drag coefficient is examined (see Table 1).

Table 1. Effect of the upstream length, L_u and the downstream length, L_d at Re = 0.1

and $\beta = 0.2$.							
$Bn = 0.01 \qquad Bn = 100$							
L_u / l	L_d / l	C_D	C_{DP}	C_D	C_{DP}		
50	20	616.97	408.27	31385	26093		
60	20	616.97	408.27	31387	26095		
70	20	616.97	408.27	31390	26098		
60	10	613.50	405.75	31386	26094		
60	20	616.97	408.27	31387	26095		
60	30	616.97	408.28	31388	26096		

Table 2. Grid independence test at Re = 40and $\beta = 0.4$.

	No of	c	<i>Bn</i> =	= 0.01	Bn =	100
Gri	d elemen	ts	C_D	C_{DP}	C_D	C_{DP}
<i>G1</i>	78000	8	.6913	7.5323	91.988	77.422
<i>G2</i>	89600	8	.6835	7.5362	91.932	77.503
<i>G3</i>	101200	8	.6783	7.5405	91.893	77.499

Table 3. Influence of regularization parameter, *m* on total drag and pressure drag coefficients at Re = 40 and $\beta = 0.4$

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	Bn =	= 10	Bn = 100		
т	C_D	C_{DP}	C_D	C_{DP}	
10^{4}	16.529	13.692	91.932	77.503	
10^{5}	16.529	13.687	92.031	76.692	
10^{6}	16.529	13.687	92.031	76.588	

Since these values varied slightly (within $\pm 0.5\%$), therefore it can be concluded that $L_u = 60$ and $L_d = 20$ are sufficient in the present study to eliminate the boundary effects. Also, in order to resolve the boundary layers at the extreme values of the parameters, a suitable computational mesh is required. Table 2 shows that the present results reported in terms of the total drag and pressure drag coefficient for grid G2and G3 differ from each other by about $\pm 1\%$; hence justifying the use of G2 in the present study. In addition, an adequate value of the regularization parameter $(m = 10^5)$ is also desirable such that any further increment in its value has no impact on the yield surfaces (see Fig. 2) and on the values of the drag coefficients. The specifics for mtest are listed in Table 3. In short, $L_{\mu} = 60$ and $L_d = 20$, grid G2 and regularization parameter, $m = 10^5$ have been found to be sufficient for the present numerical results to be fairly robust against numerical artifacts.



Figure 2. Influence of regularization parameter, m on yield surfaces at Re = 40(shaded regions represent unyielded regions).

Furthermore, in order to ensure the reliability and accuracy of the present results, comparisons were made with some benchmark numerical results available for an unconfined square cylinder in Bingham plastic fluids (Nirmalkar et al.⁴) and confined square cylinder in Newtonian fluids (Gupta et al.⁷). The present numerical results (Table 4) and (Table 5) are found to

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be within ($\pm 2\%$) and ($\pm 1\%$) with those of Nirmalkar et al.⁴ and Gupta et al.⁷ respectively. Hence, the present numerical results are regarded to be reliable to within ($\pm 1-2\%$).

Table 4. Comparison of total and pressure drag coefficients in Bingham plastic fluids for unconfined square cylinder

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Nirmalkar et al. ⁴ Present					
Re	Bn	C_D	C_{DP}	C_D	C_{DP}
0.1	1	670.89	443.83	671.17	450.13
0.1	100	31923	26681	31790	26024
40	1	2.6972	-	2.6783	-
40	100	80.020	66.293	79.847	65.243

Table 5. Comparison of total drag coefficients in Newtonian fluids for confined square cylinder, $\beta = 1/8$.

	<i>Re</i> = 5	Re = 10	Re = 40
Gupta et al. ⁷	5.549	3.511	1.864
Present	5.567	3.529	1.893

RESULTS AND DISCUSSION

The present numerical results are analyzed in terms of the morphology of yield surfaces, streamlines, pressure coefficient, shear rate contours and pressure/total drag coefficients as functions of Reynolds number, Bingham number and blockage ratio.

Streamline profiles and yield surfaces

It is customary to visualize the flow field in terms of the representative streamlines, as shown in Fig. 3. At Re = 0.1, due to low fluid inertia, the fluid elements follow the body contour but at Re = 40 wakes are observed in the rear of the square cylinder at Bn = 0.1 due to the establishment of adverse pressure gradient. In fact, as the value of Bnincreases, the fluid yield stress suppresses the tendency for flow detachment eventually leading to disappearance, of the wake altogether at high Bingham number. Furthermore, the yield stress effects also manifest in the form of yielded (unshaded regions) and unyielded regions (shaded regions) as shown in Fig. 3. It can be noted that an increase in the value of the Bingham number accentuates the extent of unyielded part whereas an increase in the value of the



Figure 3. Representative streamline profiles and yield surfaces at $\beta = 0.4$ (unshaded regions represent yielded regions and shaded regions represent unyielded regions).

Reynolds number expands the extent of the yielded region. For instance, at Bn = 10, it can be observed that vertical polar caps completely vanish at Re = 40. Not only this, the extent of yielded fluid-like region has also grown in size. Finally it can be noted that both streamlines and unyielded regions exhibit symmetry along the horizontal axis.

In the limit of $Bn \rightarrow \infty$ corresponding to the fully plastic limit, the yield surfaces and hence the drag coefficient attain a constant value.

Pressure coefficients

Further insights can be developed by examining the variation of the surface pressure coefficient, C_p which is defined as the ratio of static to dynamic pressure as:

$$C_{P} = \frac{2(p_{s} - p_{\infty})}{\rho U_{0}^{2}}$$
(6)

where, p_s is the local pressure on the surface of the cylinder and p_{∞} is the reference pressure far away from the cylinder. Fig. 4 shows that both the yield stress and the blockage ratio, β exert positive influence on the surface pressure, as



Figure 4. Variation of pressure coefficient (C_P) with Bingham number, Bn, for Re = 40.

the maximum value of the surface pressure coefficient is observed at Bn = 100 and $\beta = 0.4$. Also the magnitude of the surface pressure coefficient, C_p is maximum at the front stagnation point and minimum at the rear stagnation point because at the front stagnation point the kinetic energy of a fluid element is converted into pressure and then the fluid slowly accelerates at the expense of pressure towards the rear end.

Shear rate contours

Representative dimensionless shear rate contours are shown in Fig. 5. It can be clearly observed that shear rate is relatively high at the corners of the square cylinder and adjacent to the confining walls due to the imposed no-slip condition as compared to the unyielded regions where the shear rate is of the order of ~ $O(10^{-7})$. Furthermore the magnitude of shear rate also increases with the increasing value of blockage ratio, β . As the blockage ratio, β increases, the flow



passage becomes narrower due to which the Figure 5. Shear rate contours at Re = 40 (unshaded regions represent yielded regions and shaded regions represent unyielded regions).

shear rate increases in the regions demarcating the yielded/unyielded regions. This effect is further accentuated as fluid inertia (Re) increases.

Drag coefficients

It is the net hydrodynamic force acting on the surface of the cylinder due to normal forces, i.e., pressure drag coefficient, C_{DP} and viscous forces i.e., viscous drag coefficient, C_{DF} . The total drag coefficient is expressed as:

$$C_{D} = \frac{2F_{D}}{\rho U_{0}^{2}l} = \frac{2F_{DP}}{\rho U_{0}^{2}l} + \frac{2F_{DF}}{\rho U_{0}^{2}l} = C_{DP} + C_{DF} \quad (7)$$

where, F_D , F_{DP} and F_{DF} refer to the total drag, pressure drag and frictional drag force components respectively per unit length of the cylinder acting in the direction of flow.



Figure 6. Variation of drag coefficient (C_D) with Bingham number (Bn) at $\beta = 0.4$.



Figure 7. Variation of ratio of pressure drag coefficient to total drag coefficient (C_{DP}/C_D) with Bingham number (Bn) at $\beta = 0.4$.

Dimensional considerations suggest the individual and the total drag coefficients to be functions of the Reynolds number, Bingham number and blockage ratio. Fig. 6 and Fig. 7 show this functional relationship. It is clearly seen that the drag coefficients exhibit a positive dependence on the Bingham number and inverse dependence on the Reynolds number. Furthermore, the



pressure drag component forms a major part Figure 8. Variation of drag coefficient (C_D) with Bingham number (Bn).

of the total drag under most conditions, Fig. 7, and it also exhibits qualitatively similar behavior as the total drag coefficient. As the value of the Bingham number increases, the drag coefficient is seen to approach a limiting value which is independent of blockage ratio, β but depends on the Reynolds number Fig. 8. This is very likely due to the fully plastic flow limit. At low Bingham number depending upon the value of the Reynolds number, there is very little influence of the fluid yield stress. This behavior is clearly seen up to about $Bn \sim 0.5$ at Re = 40 whereas this limit is seen to be approached at Bn = 0.01 for Re = 0.1. This clearly reflects a balance between the viscous (augmented by yield stress) and the inertial forces. Fig. 9 shows how the contribution of pressure drag coefficient gradually increases with the increasing Bingham number and/or the blockage ratio. But eventually at large values of Bn, this ratio also approaches a constant value of about ~0.84. This behavior is qualitatively consistent with that reported in the literature (Gupta and Chhabra¹¹). Also it can be observed that blockage ratio, β has positive influence on total drag(C_D) and pressure drag coefficient (C_{DP}) which is due to the sharpening of velocity gradients and extra dissipation at the confining walls.



Figure 9. Variation of ratio of pressure drag coefficient to total drag coefficient (C_{DP}/C_D) with Bingham number (*Bn*).

Finally, the present numerical results (190 data points) on the total drag coefficient are consolidated via a simple predictive correlation as a function of the modified Reynolds number, $(Re^* = Re/(1+Bn))$ given as:

$$C_{D} = \left(39.78 \left(1 + \beta\right)^{0.8}\right) / \left(Re^{*}\right)^{0.95}$$
(8)

The resulting mean and maximum deviations are found to be 17% and 50% respectively. Only 30 data points, the deviation between the predicted and present numerical results was found to be larger than 25%. Generally large average deviations were observed at low values of

Bingham number, $0.01 \le Bn \le 1$ which could be possibly due to the regularization scheme used here. But this behavior is consistent with that for spheres in Bingham plastic fluids (Atapattu et al.¹²). CONCLUSIONS

In this study, extensive numerical results elucidating the role of blockage ratio on the steady flow of Bingham plastic fluids past a square cylinder are presented in terms of the vield surfaces. streamlines. pressure coefficient, shear rate contours and pressure/total drag coefficients over wide ranges of conditions: Reynolds number, $0.1 \le Re \le 40$; Bingham number. $0.01 \le Bn \le 100$; blockage ratio, $\beta = 0.2, 0.3$ and 0.4. The increasing Reynolds number tends to reduce the extent of unvielded while the Bingham number zones suppresses this tendency. General level of shearing also rises with the increasing blockage as does the pressure on the surface of the submerged cylinder. Under most conditions, the overall drag is dominated by the form drag. A simple predictive equation is presented which captures the effects of Reynolds number, Bingham number and blockage ratio adequately. At very high values of Bingham number, both the pressure and total drag are seen to be independent of blockage ratio. The total drag coefficient exhibits а positive dependence on the blockage ratio and the Bingham number.

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NOMENCLATURE

 C_D Drag coefficient, dimensionless

 C_{DF} Friction drag coefficient, dimensionless

 C_{DP} Pressure drag coefficient, dimensionless

 C_{P} Pressure coefficient, dimensionless

 F_D Drag force per unit length of the cylinder, $N.m^{-1}$

 F_{DF} Frictional component of drag force per unit length of the cylinder, $N.m^{-1}$

 F_{DP} Pressure component of drag force per unit length of the cylinder, $N.m^{-1}$

- H Channel height, m
- *l* Side length of square cylinder, *m*
- *m* Growth rate parameter, dimensionless
- *p* Pressure, dimensionless

 p_s Local pressure on the cylinder surface, Pa

 p_{∞} Reference pressure far away from the cylinder, *Pa*

 U_0 Inlet velocity, $m \cdot s^{-1}$

Greek symbols

- ∇ Del operator, m^{-1}
- η Apparent viscosity, dimensionless
- & Rate of strain tensor, dimensionless
- μ_{b} Plastic viscosity, *Pa.s*
- ρ Density of the fluid, kg.m⁻³
- τ Extra stress tensor, dimensionless
- τ_0 Yield stress, Pa

REFERENCES

1. Macosko, C.W. (1994), "Rheology: Principles, Measurements and Applications". New York, NY:VCH.

2. Mossaz, S., Jay, P., and Magnin. (2010), "A Criterion for the Appearance of Recirculating and Non-stationary Regimes behind a Cylinder in a Viscoplastic Fluid", *J. Non-Newt. Fluid Mech.*, **165**, 1525-1535.

3. Nirmalkar, N., Chhabra, R.P., and Poole, R.J. (2012), "On Creeping Flow of a Bingham Plastic Fluid Past a Square Cylinder", *J. Non-Newt. Fluid Mech.*, **172**, 17-30. 4. Nirmalkar, N., Chhabra, R.P., and Poole, R.J. (2013), "Laminar Forced Convection Heat Transfer from a Heated Square cylinder in a Bingham Plastic Fluid", *Int. J. Heat Mass Trans.*, **56**, 625-639.

5. Gupta, A.K., Sharma, A., Chhabra, R.P., and Eswaran, V. (2003), "Two-Dimensional Steady Flow of a Power-Law Fluid Past a Square Cylinder in a Plane Channel: Momentum and Heat Transfer Characteristics", *Ind. Eng. Chem. Res.*, **42**, 5674-5686.

6. Patel, S.A., and Chhabra, R.P. (2013). "Steady Flow of Bingham Plastic Fluids Past an Elliptical Cylinder", *J. Non-Newt. Fluid Mech.*, **202**, 32-53.

7. Bharti, R.P., Chhabra, R.P., and Eswaran, V. (2007), "Effect of Blockage on Heat Transfer from a Cylinder to Power-law Liquids", *Chem. Eng. Sci.*, **62**, 4729-41.

8. Dhiman, A.K., Chhabra, R.P., and Eswaran, V. (2008), "Steady Flow across a Confined Square Cylinder: Effects of Power-law index and Blockage Ratio", *J. Non-Newt. Fluid Mech.*, **148**, 141-50.

9. Papanastasiou, T.C. (1987). "Flow of Materials with Yield", *J. Rheol.*, **31**, 385-404

10. Mitsoulis, E. (2004). "On creeping drag flow of a viscoplastic fluid past a circular cylinder: wall effects", *Chem. Eng. Sci.*, **59**, 789-800.

11. Gupta, A.K., and Chhabra, R.P. (2013), "Spheroids in Viscoplastic fluids: Drag and Heat Transfer, *Ind. Eng. Chem. Res.*, **53**, 18943-18965.

12. Atapattu, D.D., Chhabra, R.P., and Uhlherr, P.H.T. (1995), "Creeping Sphere Motion in Herschel-Bulkley Fluids: Flow Field and Drag", *J. Non-Newt. Fluid Mech.*, **59**, 245-265.