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Viscosity of Suspensions and Finite Size Effects

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ABSTRACT

The viscosity of particle suspensions depend on the volume fraction of particles. This was demonstrated by Einstein[1,2] for dilute suspensions and later extended to less dilute systems by Batchelor[3] and Batchelor and Green[4]. Krieger and Dougherty[5] followed a different approach to analyse concentrated suspensions. They combined the result of Einstein with a self-consistent field approach and obtained a very useful expression for the viscosity as a function of the solvent viscosity (η_s) and the volume fraction of particles (φ) as well as a maximum packing parameter (φ_m). The maximum packing parameter φ_m, though physically sound, was introduced in a very heuristic way.

In this work we consider a model for the viscosity of particle suspensions which takes into account finite size effects and a clear physical interpretation of the packing parameter φ_m. In the limiting case of dilute suspensions the result of Einstein is recovered whereas the Krieger-Dougherty expression is approximately found for very small particles.

Data reported by de Kruiff et al.[6] were compared to predictions by the models. We find that the inclusion of finite size effects improves the model description for the viscosity.

INTRODUCTION

The viscosity of particle suspensions depends on the volume fraction of suspended particles. Based on viscous dissipation by the flow around a single isolated non-deformable sphere, Einstein[1,2] calculated the change in viscosity η to first order in the volume fraction φ.

$$\eta=\eta_s(1+[\eta]\varphi) \qquad (1)$$

Here, η_s is the solvent viscosity and the intrinsic viscosity $[\eta]=5/2$. In general the intrinsic viscosity is defined by Eq. 1 in the limit $\varphi\to 0$ and depends on the geometry and deformability of the suspended particles. Hydrodynamic interactions between two spheres were taken into account by Batchelor[3] and Batchelor and Green[4] resulting in an expression for the viscosity to second order in the volume fraction. A different approach was taken by Krieger and Dougherty[5] as described in the section below.

KRIEGER-DOUGHERTY MODEL

In the following we describe the basic theory leading to the simplest version of the Krieger-Dougherty equation. This model does not include a maximum packing fraction for particles.

We consider a volume composed of solvent and particles so that the total volume equals the sum of solvent and particle volumes V_s and nV_p respectively. Here, V_p

denotes the volume of a single particle and n is the number of such particles.

The volume fraction of particles is then calculated from

$$\varphi = nV_p/(V_s+nV_p) \quad (2)$$

The change in the volume fraction by adding particles is described by

$$d\varphi/dV_p = \varphi(1-\varphi)/V_p \quad (3)$$

We now assume that the solvent including n particles can be considered a continuum fluid. Thus we assume Einstein's expression for the viscosity of dilute suspensions can be used when adding particle number n+1. We obtain,

$$\eta_{n+1} = \eta_n + \eta_n[\eta]\varphi_n^* \quad (4)$$

where φ_n^* is the single particle volume fraction

$$\varphi_n^* = V_p/(V_s+(n+1)V_p) \quad (5)$$

Inserting φ_n^* in the expression for η_{n+1} we obtain

$$(\eta_{n+1} - \eta_n)/((n+1)V_p - nV_p) = (\eta_n[\eta])/(V_s+(n+1)V_p) \quad (6)$$

Letting $V_p \rightarrow 0$ while keeping the total volume of particles nV_p constant we find the continuous limit

$$d\eta/dV_p = ([\eta]\eta)/(V_s+nV_p) \quad (7)$$

and further

$$d\eta/d\varphi = [\eta]\eta/(1-\phi) \quad (8)$$

A solution is then found by integration. We obtain

$$\eta(\varphi) = \eta_s(1-\varphi)^{-[\eta]} \quad (9)$$

which describes the viscosity in a suspension with very small particles without a maximum packing fraction. As the volume fraction of particles increases a limit is reached where no further particles can be added to a suspension without also adding more solvent or allowing the formation of air voids. To further develop the theory a maximum packing fraction φ_m is therefore required in the model.

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